

Japanese Education and Lesson Study: An Overview

Section 1.1: “How is Lesson Study Implemented?”

Takuya Baba

Introduction: Lesson Study, currently a topic of worldwide attention, refers to a process in which teachers progressively strive to improve their teaching methods by working with other teachers to examine and critique one another’s teaching techniques. First developed as an educational practice in the Meiji period of Japan, Lesson Study functions as a means of enabling teachers to develop and study their own teaching practices. It is this function to which its international attention can be attributed.

1. The Process of Lesson Study

Lesson Study consists of *preparation, actual class, and class review sessions* in Japanese “kyozai kenkyu”, “koukai/kenkyu jyugyo” and “jyugyo kentoukai”. The process of transforming a planned curriculum, such as that found in National Course of Study or textbooks, into a curriculum that can be implemented in the classroom is referred to as “preparation”, the first stage of the Lesson Study process. This process begins with finding and selecting materials relevant to the purpose of the class, and is then followed by refining the class design based on the actual needs of the students and tying all of this information together into a lesson plan. The significance of Lesson Study is that all of these processes are performed in collaboration with other teachers. A class (Photo 1) is then taught based on the teaching plan devised. The class is observed by many teachers, who are sometimes joined by university instructors and supervisors from the board of education, and a review session is held for all observers after the class. This process is shown in Figure 1. Steps (1) to (4) comprise the first stage, and the results of the evaluation in step (4) are utilized in the second stage, steps (5) to (7), to refine the class. The

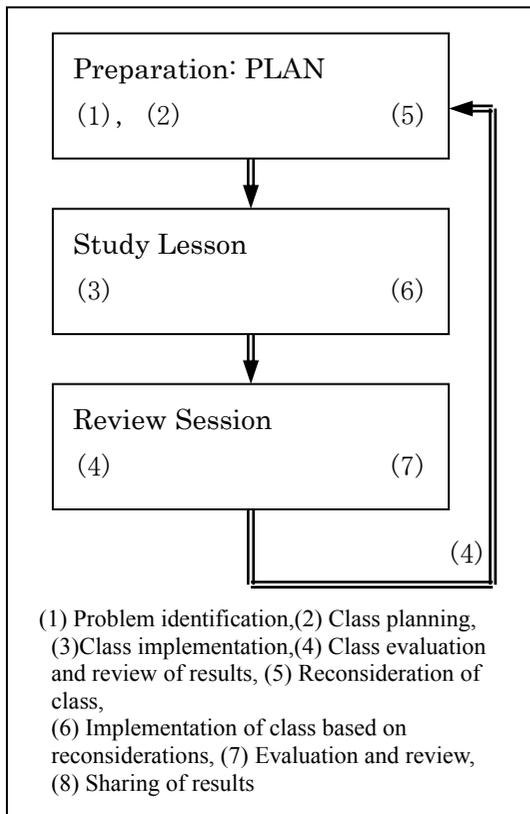


Photo 1: A study lesson class with observers

Figure 1: Flowchart of Pedagogical Training (Stigler&Hiebert,1999)

thoughts of individual teachers, improvement of the level of teaching techniques, and the breadth of the network among teachers all come into play in this process.

2. Class and Discussion Topics: A Case Study

The following details a case study of a research classroom (Figure 2) to be evaluated. Given the introduction of new perspectives on scholastic ability, the current National Course of Study emphasizes the ability to think proactively and autonomously. For example, the question of how to incorporate the cultivation of proactive thinking in subject-based learning is an important practical study theme. Preparation is performed on this theme and a lesson plan is prepared. In some cases teachers are asked to develop an index for measuring the student's level of achievement by using specific numerical scores.

In the review session following the class, the instructor gives brief introductions and explains the purpose of the class. Based on the teaching plan distributed ahead of time, concepts on teaching materials and characteristics/status of the students are described in accordance with each stage of the class. Also, the rationales for each problem and activity conducted in the class are explained. Then each participant draws upon their own teaching experiences to express opinions and ask questions about the problems given in the class and teacher's instructive role, as well as about the students' utterances and learning activities. The purpose of this review session is to explore ways to improve the class by analyzing any disparities between the original goals established for the classroom or the plans developed to achieve those goals, and what is actually happening in the classroom (Figure 3). Of significant interest is the ability of this process to facilitate the discovery of new problems or issues that had not initially been noticed during the class.

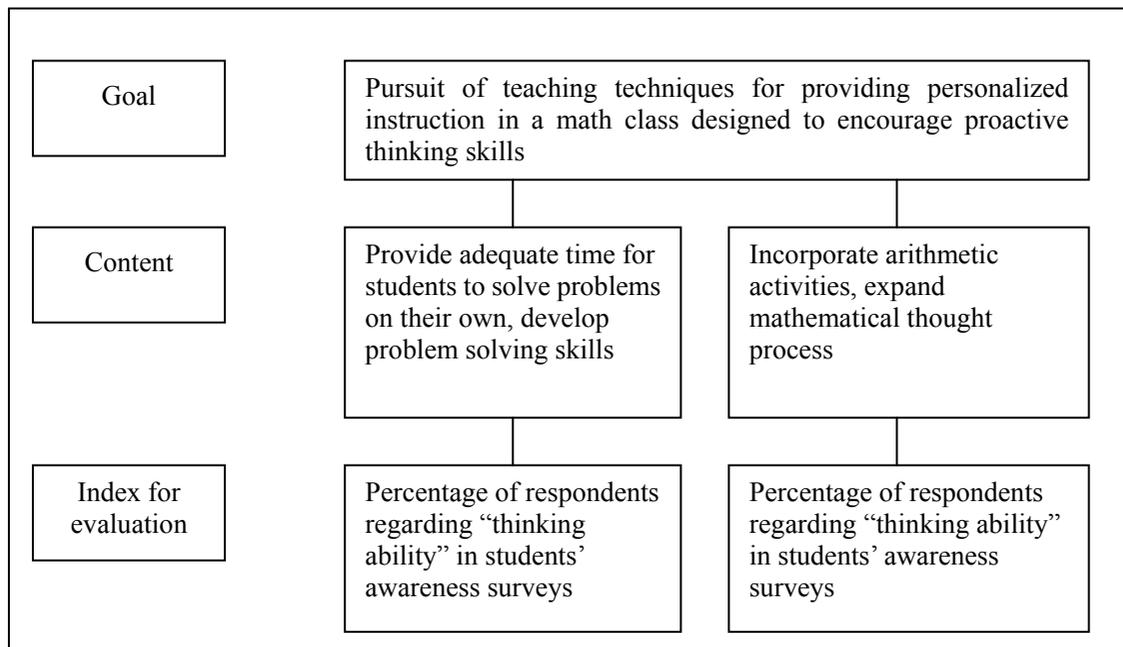


Figure 2: An Example of Topics for Study Lesson.
 (Edited excerpt from www.pref.hiroshima.jp/kyouiku/hotline/)

Types of Lesson Study Formats

Conducting Lesson Study involves a large number of teachers, but it can be done on many different scales and in varying formats (Table 1). The most common format is in-school training conducted at the school level. Lesson Study is conducted by developing an annual pedagogical theme and forming teams for each subject and grade. The classes themselves are actually taught by their regular teachers, but the process of working together empowers individual teachers in their classes and also fosters good relationships between colleagues who teach at the same school.

Other formats include Lesson Study conducted by groups of teachers on a voluntary basis and Lesson Study sessions hosted by teachers' unions and academic societies. Given the multitude of formats in which these efforts are being conducted, it appears that Lesson Study has taken root in Japan's educational culture in the true sense and that it is exerting a significant impact on the quality of education.

Finally, we will look at the systems that exist to provide public support for Lesson Study efforts. Teachers are legally and socially expected to continue upgrading their skills even after they have attained a teaching position. Public training workshops (new teacher training, annual training, etc.) are designed to give teachers opportunities to work on upgrading their skills based on their experience. Some workshops are mandatory while others are voluntary. Even in these training workshops, Lesson Study is employed as a strategy for cultivating teaching skills. When thinking about the global issue of improving the quality of education, Lesson Study, which has a dialectical relationship with the theories and practices employed on the front lines of education, must be versatile enough to be applicable beyond the Japanese context.

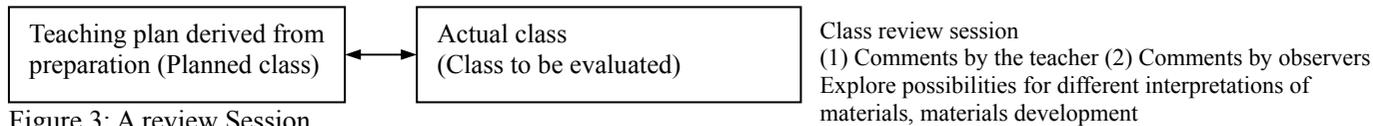


Figure 3: A review Session

Table 1: Lesson Study Formats in Japan (Ikeda et al., 2002, p. 28)

	Scale of participation	Main sponsor(s)
1	Individual schools	Public school principals and teachers (In-school training)
2	Study groups at the prefectural, municipal, and ward levels	Public school teachers
3	Study groups at the prefectural, municipal, and ward levels	Board of education, education center
4	Nationwide	Principals and teachers at schools affiliated with universities
5	Prefectural level, nationwide	Private institutions (academic societies, corporations, etc.)

Example: Hiroshima Prefecture
 Implemented by: Board of Education, Education Center

- Compulsory training (1st and 2nd years, 6th year, 11th year)
- Voluntary training

Purpose: To improve teaching skills and problem-solving skills

Figure 4: Public training system (Edited excerpt from <http://pfrq3.hiroshima-c.ed.jp/>)

A Brief History of Mathematics Lesson Study in Japan

Section 2.1: “Where did Lesson Study Begin, and How Far Has It Come?”

Masami Isoda

Lesson Study began in the late 19th century with class visits designed to allow the whole classroom instruction.

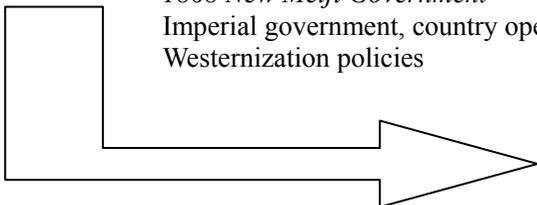
1. From Individualized Instruction to whole classroom instruction: Studying Teaching Methods

Under the seclusion policies and class system that characterized the Edo period for about 260 years prior to the installation of the new Meiji government in 1868, literacy (and numeracy) education was available to commoners through *terakoya*, or temple schools, that had opened up autonomously around the country. Commerce thrived and the class system gradually collapsed during this period of seclusion, and by the late Edo period, individual knowledge and skills were highly regarded in the recruitment of workers. Due to the widespread emergence of temple schools, to which parents could voluntarily send their children, the literacy rate at the end of the Edo period was 43% among males and 10% among females, even then making Japan one of the most educated countries in the world. Individualized instruction was the common teaching method employed.

In 1872, the Meiji government issued the Education Code and at the same time established a teachers' school (normal school) in Tokyo (forebear to University of Tsukuba). With the goal of disseminating Western scholarship, the government invited foreign



1868 New Meiji Government
Imperial government, country opening,
Westernization policies



1872 Education Code issued, teachers' schools
established

How is teaching done in a classroom?
How does group instruction work?



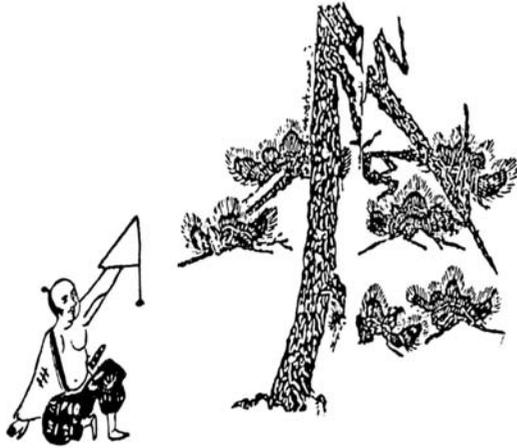
Pestalozzi: Experience-oriented
teaching methods

Figure 1: Shift from the curriculum and teaching methods of the *terakoya* (temple schools) to those of new types of schools.

teachers to teach Western subjects. The foreign teachers introduced the concept of whole classroom instruction, a style then still rare even in the West, into the teachers' school (Figure 1). The Japanese teachers and students, who were familiar only with the individualized instruction model in which subjects were taught individually based on the academic abilities of the student, learned not only the contents of the subject, but also methods of teaching by observing their teachers' behavior.

Textbooks created by foreign teachers at the teachers' school contained drawings of students raising their hands to answer questions posed by the teacher, as shown in Figure 2. It contained the question "How many students are raising their hands?" This foreign teacher wrote a textbook that teaches instruction methods as well as mathematics at the same time. The group instruction model implemented at the teachers' school in Tokyo spread to other teachers' schools around the country. Due to financial difficulties, the new government closed down all the teachers' schools except the one in Tokyo around 1880.

Nonetheless, in the decade while the schools were open, the practice of group instruction was disseminated around the country by graduates of the teachers' schools and by scroll pictures (Figure 1, right) and textbooks (Figure 2, right).



“How tall is the tree?”

Illustration from *Jinkoki*, a mathematics textbook from the Edo period.



“How many people are raising their hands?”

Illustration from an elementary mathematics textbook in 1873.

Figure 2: From textbooks (left) that allowed students to study numeracy at their own discretion, depending on their needs, to textbooks (right) designed to allow students/teachers to simultaneously study leaning/teaching methods.

2. Dissemination of the Lesson Study Practices through the Elementary School Attached to the Tokyo Teachers' School.

In the 1880s, study on group instruction and its dissemination reached new heights as overseas study missions began returning to Japan. Mission delegates, who had been teachers at the teachers' school before their departures, were invited to become teachers at the elementary school attached to the teachers' school after their return, and a book on the Pestalozzi's teaching method was published. Even back then, this book contained comments on teaching materials, as well as instructions for conducting class observation and holding critique sessions. At the instruction of the Ministry of Education, these teaching methods were implemented throughout Japan as a model. Open classes, the origin of study lessons, were held to encourage the proposal of new teaching methods and teaching curricula, producing the first interactive Lesson Study groups initiated by the government. Figure 3 shows one of the national teachers' training conferences, which have been held since the Meiji period.

3. Development and Dissemination of Teaching Methods Learned through Lesson Study

As the country grew wealthier, it became possible for anyone to attend and graduate from elementary school. In the 1920s, new teaching methods based on the educational philosophy of scholars like John Dewey launched an era in which non-government-attached-school teachers began proposing their own teaching methods. At this time, a new teaching method was proposed for enhancing peer learning (see Figure 4). It allowed students to come up with their own study questions, discuss with one another whose question they wanted to research, and then go about researching the selected question. This set the stage for the emergence of teaching methods that focus on problem-solving, which today are globally recognized

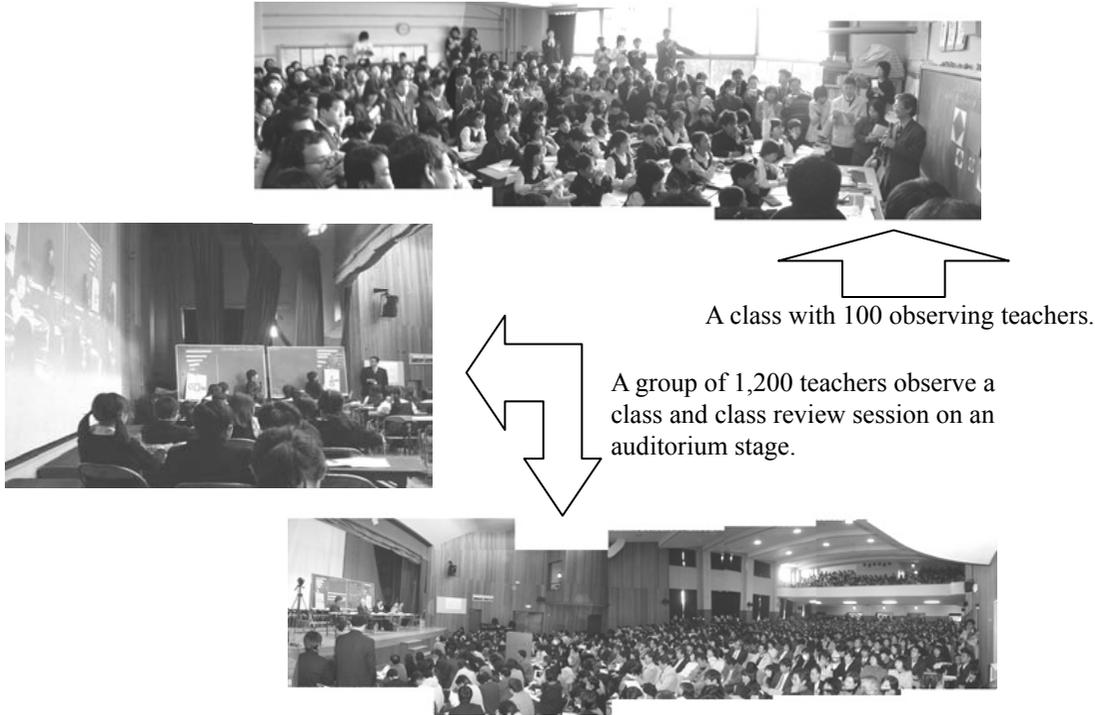


Figure 3: National Training Conference for Teachers at the Elementary School Attached to the University of Tsukuba, held since the Meiji period.

as models of constructivist teaching. Teachers' unions were launched after World War II, and Lesson Study by involved teachers led to heated debates. These classes also came to be used for launching futile ideological opposition. Teaching methods focused on problem-solving, which recognized the limitations of what already known and tried to produce new knowledge, were able to achieve success in spite of having to overcome the conflicts and other challenges. This was possible because visiting teachers were exposed to classes conducted for observation, and were impressed by seeing the students learning by themselves through problem solving.

Now, problem-solving approach is well known as a major way of teaching mathematics in Japan.

4. How Japanese Lesson Studies and Approaches are developed and known: A case of Open Approach

After 1943, Japanese National Secondary Mathematics Textbooks integrated different mathematical subjects into a single subject. Shigeru Shimada was an author. The textbooks were written with a focus on the processes of Mathematization and Open-ended problem solving. In the later 1960s, Shimada developed the research project of evaluation with Open-ended problems. In 1970s, the project had expanded to the Lesson Study projects for developing new teaching approaches; currently we distinguish them as 'Processes are open (various solving ways)', 'Ends are open (various answers against an open-ended problem)', and 'Problems are open (changing and developing problems from a problem)'. Later, Nobuhiko Nohda integrated them as the teaching method of 'Open Approach'. In the 1980s, Jerry Becker, Tatsuro Miwa and others began the collaborative study on problem solving between USA and Japan. On the contribution by Jerry Becker with co-researchers, as well as other simultaneous research movements, these are well known in the USA with classroom Lesson Study activities (See Chapter 5, Case 4 by Yoshihiko Hashimoto in this volume and Jerry Becker & Shigeru Shimada, 1997).



Children devise with their own study questions and write them on small chalkboards in the school hallway.

The boards are hung in the classroom to present the proposed ideas.



Figure 4: Study is conducted on how to teach students to develop their own study questions at the elementary school attached to Nara Women's higher normal school around 1920.

Official In-Service Teacher Training System

Section 3.1: How is In-Service Teacher Training Conducted in Japan?

Kazuyoshi Okubo

1. The In-Service Teacher Training System

Japanese education policies are aimed at developing people of well-rounded character with the ability to learn and think on their own, make decisions, act independently, solve problems, collaborating well with others, and to have compassion and sensitivity toward others. Achieving these goals hinges on the capabilities of teachers. The process of educating, hiring and training teachers presents an opportunity to improve teachers' capabilities (Figure 1). Training can be divided into three types, based on their relationship to the teachers' official duties. The first is training conducted by the government as part of the teachers' duties. The second is training that is independently conducted outside school during working hours and recognized under the Law for Special Regulations Concerning Educational Public Service Personnel. The third is voluntary training held outside working hours. Boards of education and universities with teacher certification programs support independent study groups in their local areas and offer venues for work-time training and voluntary training. Work-time and voluntary training programs tend to be organized by the teachers themselves and have the characteristics of "research."

2. Training Organized by the Government

The government holds planned, organized, systematic training programs. To achieve its educational goals, the national government provides financial support for training programs, and in each prefecture conducts training for teacher leaders and training to address pressing issues relating to school education (Figure 2). The content of the training is determined by officers (as well as councils,

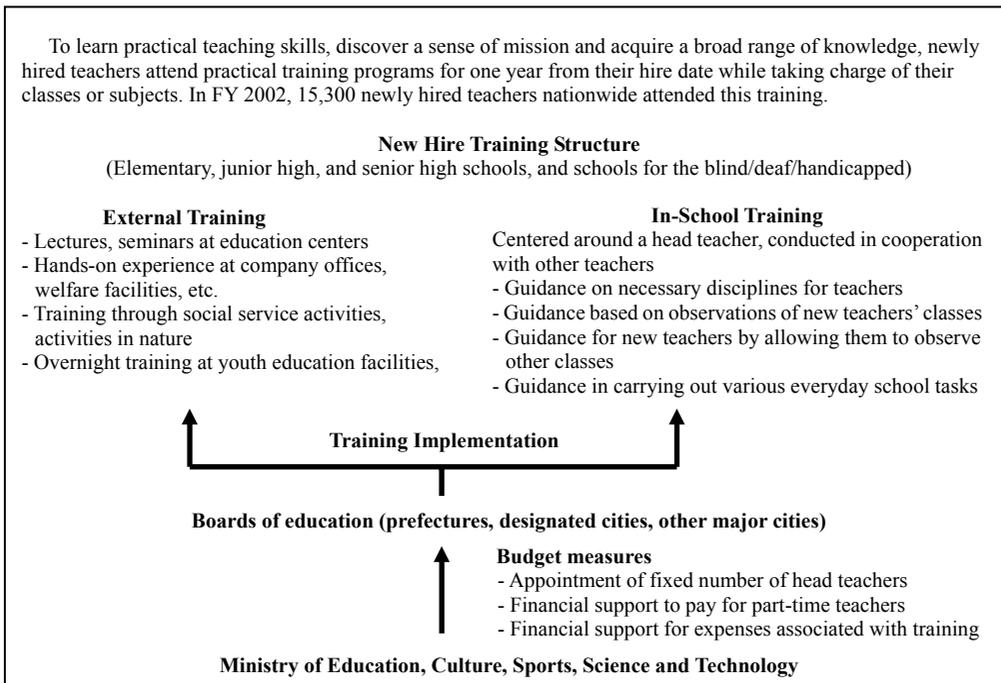


Figure 1: New Hire Training

etc.) assigned to research subjects. Supervisors of the prefectural education centers and other educational institutions plan and implement government training programs. Training for new teachers aims to impart practical teaching skills and a sense of mission, as well as to enable teachers to acquire a broad range of knowledge. Training sessions are attended for one year from the teachers' date of hire (see Figure 3). There is also 10th year training. It aims to improve teaching skills in various subjects and is adapted to the capabilities and needs of individual teachers. In addition, the education centers also provide special training classes to provide instruction in various subjects for teachers who want additional training. Special training courses are also held to teach new contents of a subject when education reforms are implemented, and to teach new evaluation methods when evaluation standards are revised.

3. In-School Training (Research)

In-school training forms the central role in the development of teaching skills. Administrators and teachers alike refer to training as “research.” In particular, administrators promote in-school research to enable groups of teachers to systematically work with one another to improve their teaching skills by establishing a training committee and assigning training leaders for each grade. The training committee decides on the training topics the school will address for the following year and establishes a training plan for enhancing teachers' knowledge of those topics. For example, the committee determines who will conduct the classes that will be observed for evaluation each month. The assigned teacher then works either with a group of teachers of the same grade or with relevant teachers in other grades to plan and implement the class. Other interested teachers observe this class, and the person who is assigned to teach the observed class next uses the feedback obtained to plan their lesson. To ensure that the research goes smoothly, the training committee may organize training sessions led by university professors. The administrators provide external training opportunities to each teacher, based on the plan developed by the training committee. Results of in-school training are verified by what students are seen doing during classroom visits. Public

Financial Measures for Retaining Teachers

To ensure the retention of the most outstanding teachers, special measures even beyond those in place for general government workers, have been taken with regard to public school teachers' salaries. Half of the salaries of public elementary and junior high school teachers is paid for by the prefectural governments, while the other half is paid for by the national government (pursuant to the system where the national treasury pays a share of compulsory education expenditures).

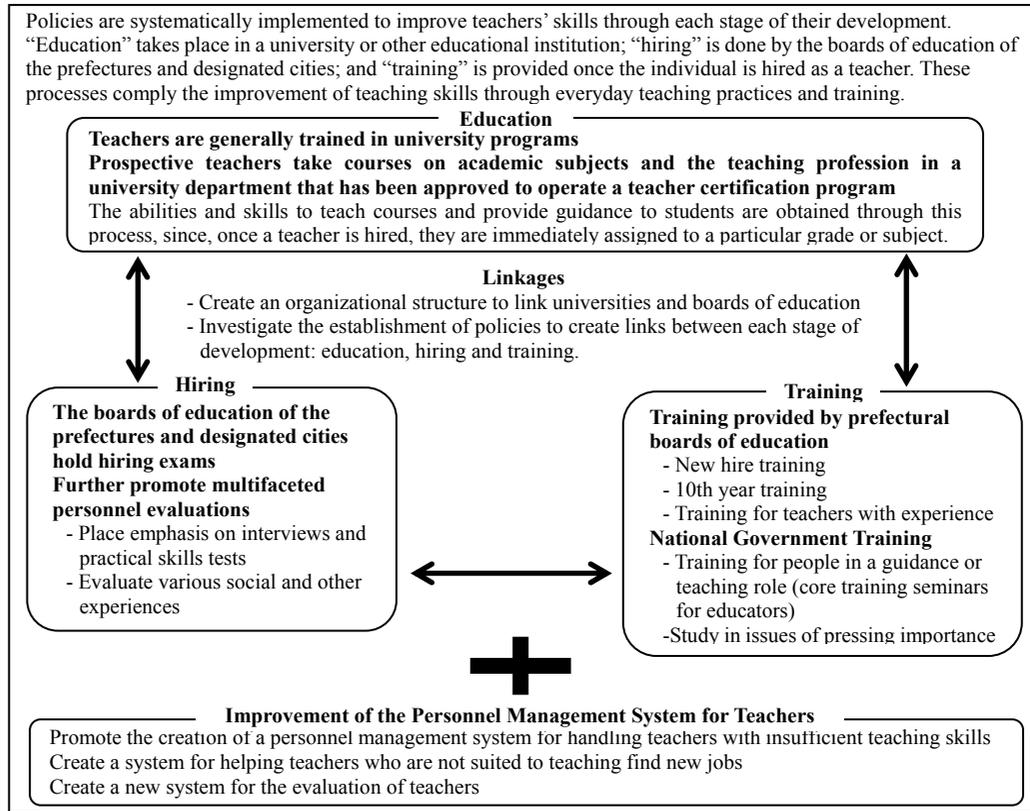


Figure 2: Systematic Structure Throughout Education, Hiring and Training

research meetings are occasionally held to provide teachers with a forum for presenting the achievements of their training.

4. Voluntary Training and Work-time Training

Voluntary and work-time training programs that do not receive financial support are carried out through teachers' voluntary efforts. One way they can do this is to participate in public research meetings held by other schools. Observing well-conducted classes and learning stimulates teachers and helps them develop better classes. If government training is a top-down approach to training, these kinds of training represent a teacher-instigated bottom-up approach. Achievements of training are shared with the public through national conferences of academic societies (such as the Japan Society of Mathematical Education) and at meetings for presenting research that are held by mathematical education associations at the prefectural and municipal levels.

5. A Wider Perspective

These three categories – Training organized by the government, In-school training (Research), and Voluntary training and Work-time training – reflect either a management or a perspective based on financial responsibility for Lesson Study activities. While this is a useful perspective, Lesson Study does need to be seen from the perspective of teachers themselves. Teachers see Lesson Study as an integral part of their professional life regardless of who is organizing it, and as something to be enjoyed.

Teachers in Japan expect to participate in Lesson Study activities at all stages of their careers and make different contributions as they become more experienced or take on higher responsibilities in schools. During their careers, they are expected to deepen through their own practice and participation in Lesson Study their knowledge of students' development and how to cultivate rich learning. In addition, many other agencies participate in and support Lesson Study activities – universities, publishers and so on. Their roles have been discussed chapter 4 in this book and in the Preface.

- Training -

1 Enhancement of the Systematic Training Structure

To fulfill their responsibilities, educators are required to undergo continuous training. The boards of education of prefectures, designated cities and other major cities are required to offer planned training sessions. A systematic training structure has been developed for different types of training starting with new hire training.

The national government provides support for the training activities implemented by the prefectural governments and also holds training sessions for educators who are in leadership roles and seminars on pressing topics relevant to school education at the National Center for Teachers' Development.

Teacher Training Structure		1st yr.	5th yr.	10th yr.	15th yr.	20th yr.	25th yr.	30th yr.	35th yr.
National level training	Leadership training	<p>Core Training Courses for Educators</p> <p>For leader teachers for principals and vice-principals</p> <p>Seminars on giving guidance to students on courses after graduation, seminars for teachers of new industrial technologies</p> <p>Overseas delegations of teachers</p> <p>Short-term overseas delegations</p> <p>Overseas training for young teachers Japan-U.S. exchange programs Delegations of young teachers to the U.S.</p>							
		Seminars on pressing topics	Training for teachers promoting the use of information technologies in education, training sessions on education to prevent AIDS and drug abuse						
Training implemented by boards of education of prefectures, designated cities, and core cities		New hire training		10th year training		<p>Training for teachers with experience</p> <p>5th year training 20th year training</p> <p>Training for student guidance supervisors</p> <p>Training for new head teacher of instruction</p> <p>Vice-principal training</p> <p>Principal training</p> <p>Long-term external training at a private company</p> <p>Specialized training in subject teaching, student guidance (held by education centers)</p> <p>In addition to these, voluntary training sessions are also held at municipal board of education facilities and in schools.</p>			

Figure 3: Improve Teaching Skills, Expand Horizons

Note: Figures 1-3 are excerpts from the "Pursuit of Teachers with Great Characters" article on the website of the Ministry of Education, Culture, Sports, Science and Technology.
<http://www.mext.go.jp>

Mathematics Curriculum and Way of Implementation

Section 4.1: How Has Mathematics Education Changed in Japan?

Eizo Nagasaki

Mathematics education in Japan since the Meiji period, when the shift was made to a modern education system, can be divided into five phases from a curriculum development perspective:

- Phase I: Assimilation of mathematics education from western Europe (1860s to 1930s)
- Phase II: Formation of Japan's own mathematics education (1930s to 1940s)
- Phase III: Establishment of the foundations of Japanese mathematics education (1950s to 1960s)
- Phase IV: Modernization of mathematics education based on international trends (1960s to 1970s)
- Phase V: Development of mathematics education suitable for students (late 1970s forward)

Phase I began with the use of mathematics curricula from other countries and translated textbooks. Ideas for mathematics reforms proposed by such foreign mathematicians as John Perry and Felix Klein were later compiled into the "Mathematics Education Improvement Campaign" and these led to improvements in Japanese mathematics education.

In Phase II, there was a shift in mathematics education from the perspective of "teaching mathematics in a way that makes it easy to understand" to the perspective of "teaching children to create

Table 1: Changes in Mathematics Education in Japan and Around the World (continue to page 25)

Trends in Japanese mathematics education	World trends as they pertain to Japan
<p>I. Assimilation of mathematics education from western Europe 1860s-1930s 1868 Meiji Restoration. 1872 Regulations for the National Courses of Study for Elementary and Junior High Schools (first national curricular standards). 1905 A textbook for elementary schools, <i>Ordinary Elementary School Arithmetic</i> (black cover), was first used (the first indigenous Japanese elementary school mathematics textbook). 1919 The mathematical Association of Japan for Secondary Education (precursor to the Japan Society of Mathematical Education). 1925 Metric system begins to be incorporated into mathematics textbooks. 1931 National courses of study for junior high school mathematics are revised.</p>	<p>1901 Perry lectures “Teaching of Mathematics” in England. 1902 Secondary education system reforms are implemented in France. 1902 R. L. Moore lectures on the foundations of mathematics in USA. 1904 Felix Klein lectures “Mathematics Education in High School” in Germany. 1912 Japan gives a presentation at a meeting of the International Commission on the teaching of Mathematics at the 5th International Congress of Mathematicians (ICM).</p>
<p>II. Formation of Japan's own mathematics education 1930s-1940s 1935 Elementary schools begin to use <i>Ordinary Elementary School Arithmetic</i> (green cover), a textbook for elementary schools. 1940 Committee for Reorganizing Syllabi of Mathematics is established. 1942 National Courses of Study for junior high schools and girls’ high schools are mathematics revised. 1942 Schools begin <i>First and Second Categories in Mathematics</i>, a junior high school textbook.</p>	<p>1936 Japan gives a presentation at a meeting of the International Commission on the teaching of Mathematics at the 10th International Congress of Mathematicians (ICM). 1938 American Progressive Education Association publishes <i>Mathematics in General Education</i>.</p>
<p>III. Establishment of foundations of Japanese mathematics education 1950s-1960s 1951 The National Course of Study (Tentative) is published for elementary, junior high, and senior high schools (learning by the method) 1953 JSME holds the Joint Conference on Curricula of Elementary, Junior High, and High Schools. 1955-60 The National Course of Study for elementary, junior high, and senior</p>	<p>1951 University of Illinois Committee on School Mathematics (UICSM) is launched. 1956 Japan participates in the International Conference on Public Education, Mathematics Education in Higher Education, under</p>

mathematics." ¹ However, this was never able to be fully implemented because of the war.

Phase III began after the war with “Learning by Unit”, which emphasized the social utility of mathematics², but a movement by the Japan Society of Mathematical Education (JSME) later led to a shift toward systematic learning, which emphasized the structures of mathematics. It was this shift that firmly established the foundations of Japanese mathematics education.

In Phase IV, Japan took lessons from the modernization of mathematics education in other countries and, spurred by a movement of the JSME, introduced modern concepts and approaches into Japanese mathematics education with the terminologies for developing mathematical thinking.

In Phase V, international cooperation in the form of international studies and conferences has flourished, and efforts are being taken to develop mathematics education designed to meet the needs of primary and secondary school students.

¹ Editors’ note: Mathematical activities were enhanced from this age on the national curriculum.

² Editors’ note: It was the second influence of the progressivism from US.

Table 1 (continued)

<p>high schools is issued (systematic learning).</p>	<p>UNESCO.</p>
<p>IV. Modernization of mathematics education based on international trends 1960s-1970s</p> <p>1961 JSME publishes the Research Journal of Mathematical Education</p> <p>1963 JSME launches the Mathematics Curriculum Research Committee.</p> <p>1966 JSME publishes <i>The Modernization of Mathematics Education</i>.</p> <p>1968-70 The National Course of Study for elementary, junior high, and senior high schools is issued (modernization).</p>	<p>1958 School Mathematics Study Group (SMSG) is launched in the USA.</p> <p>1959 Organization for European Economic Cooperation (OEEC) holds a seminar on "New Approaches to Mathematics."</p> <p>1964 International Association for the Evaluation of Educational Achievement (IEA) holds its First International Mathematics Study.</p> <p>1969 The 1st International Congress on Mathematical Education (ICME) is held (Lyon, France).</p> <p>1974 ICMI-JSME Regional Conference on Mathematical Education is held.</p>
<p>V. Development of mathematics education suitable for students Late 1970s and forward</p> <p>1977-78 The National Course of Study for elementary, junior high, and senior high schools is issued (fundamentals/basics).</p> <p>1989 The National Course of Study for elementary, junior high, and senior high schools is issued (internationalization/computerization/individualization).</p> <p>1998-99 National Course of Study for elementary, junior high, and senior high schools is issued (zest for life).</p> <p>2002 The National Institute for Educational Policy Research conducts a Study of Curriculum Implementation Status among elementary, junior high, and senior high school students.</p>	<p>1980 National Council of Teachers of Mathematics (NCTM), USA, announces its "Agenda for Action" (emphasizing problem solving).</p> <p>1980 IEA holds its Second International Mathematics Study.</p> <p>1983 ICMI-JSME Regional Conference on Mathematical Education is held.</p> <p>1990- NCTM, USA, announces "Curriculum and Evaluation Standards for School Mathematics."</p> <p>2000- The 9th International Congress on Mathematical Education (ICME) is held (Tokyo).</p> <p>2000- Organization for Economic Co-operation and Development (OECD) implements Programme for International Student Assessment (PISA).</p>

Section 4.2: How Have the Goals of the Mathematics Curriculum Changed?

Eizo Nagasaki

The national curricular standards for elementary, junior high, and senior high schools in Japan in the postwar period are stipulated in the National Course of Study (*gakusyu shidou youryo*). The changes in the goals of mathematics education can be understood by looking at the mathematics goals stated in the National Course of Study in each decade. The goals are described in four evaluation perspectives in the Permanent Cumulative Record (*gakusyu shidou youroku*) that provides national standards for developing student evaluations' criteria. The table below shows features of the curriculum, the mathematics goals of the National Course of Study for elementary schools, and the criteria of mathematics evaluations in the Elementary School Permanent Cumulative Record.

The goals of the mathematics curriculum in Japan have included the "cultivation of mathematical thinking" since the 1960s.¹ Other mathematics goals in each decade are as follows:

1950s: Education in mathematics problem solving skills to solve social problems

1960s: Understanding of mathematics concepts

1970s: Cultivation of the ability to think from integrating and developing point of view.²

¹ Editors' note: The cultivation of mathematical thinking has being enhanced for clarifying the quality of mathematical activities which were enhanced in 1950s.

² Editors' note: It used to be focusing on the restructuring nature of mathematical development with the invariant terminologies of mathematical

Table 1: Changing Goals of the Mathematics Curriculum (continue to page 29)

Decade	Major features of the mathematics curriculum	Mathematics goals of the National Course of Study for elementary schools	Criteria of mathematics evaluations in the Elementary School Cumulative Guideline Reports
1950s	<p>Learning by the unit method Curriculum consists of mathematics concepts and methods for understanding and finding mathematical solutions to social problems and problems concerning everyday issues.</p>	<p>(1) It is important to improve skills that enable students to solve the problems that arise in everyday life on their own, as necessary. (2) It is important for children to have the desire to make their lives better through quantitative processing. (Remainder omitted.)</p>	<p>Interest/attitude toward numbers and quantity Quantitative insight Logical thinking Calculation and measurement skills</p>
1960s	<p>Systematic learning Curriculum consists of mathematics concepts and mathematical thinking.</p>	<p>1. Enable children to understand the basic concepts and principles of numbers and figures, and foster more advanced mathematical approaches and ways of processing information. 2. Teach basic knowledge pertaining to numbers and figures as well as proficiency in basic skills, and enable children to use those skills accurately and efficiently for a given purpose. (Remainder omitted.)</p>	<p>Interest in numbers and quantity Mathematical thinking Understanding of terms and symbols Calculation skills</p>
1970s	<p>Modernization Curriculum incorporates modern mathematics approaches, concepts, and content.</p>	<p>Cultivate skills and attitudes that will allow students to have a mathematical understanding of everyday events, think coherently, and to observe and process information in an integrated and developmental manner. (Remainder omitted.)</p>	<p>Knowledge/understanding Skills Mathematical thinking</p>

1980s: Acquisition of basic mathematics knowledge and skills

1990s: Appreciation of the meaning of mathematics

2000s: Getting enjoyment from mathematics activities

Based on this framework, one could argue that the goals of mathematics education in Japan emphasized the social need for mathematics in the 1950s, the mathematical need for mathematics in the 1960s and 1970s, and the needs of children since the 1980s.

The more recent goals of mathematics education in Japan, analytically speaking, have consisted of four main components: interest / eagerness / attitude, mathematical thinking, expression / processing, and knowledge / understanding.

Editors' Note: Further readings

Editorial Department of Japan Society of Mathematical Education (JSME) published the following books in English on mathematics education in 2000: 1) Mathematics Education in Japan during the Fifty-five Years since the War: Looking towards the 21st Century. 2) Exploring Elementary School Mathematics Education of Japan in the 21st Century; Based on Practical Studies in the 1990s. 3) Exploring Secondary School Mathematics Education of Japan in the 21st Century; Based on Practical Studies in the 1990s. 4) Mathematics Program in Japan; Elementary, Lower Secondary & Upper Secondary Schools.

Further information:

http://www.sme.or.jp/e_index.html

thinking. The ideas originated in Japan by World War II, and are not the same but not so far from *Mathematization* by H. Freudenthal (1968/1973).

Table 1 (continued)

1980s	<p>Fundamentals/basics Curriculum stripped of modern mathematics concepts and content that were introduced during the period of modernization.</p>	<p>Impart fundamental knowledge and skills pertaining to numbers and figures, cultivate skills and attitudes that will allow students to have a mathematical understanding of everyday events, think and process information coherently.</p>	<p>Knowledge/ understanding Skills Mathematical thinking Interest/attitude toward numbers and figures</p>
1990s	<p>Internationalization/computerization/individualization Curriculum actively incorporates computer and elective learning.</p>	<p>Impart fundamental knowledge and skills pertaining to numbers and figures, cultivate skills that will allow students to have an outlook on everyday events and to think coherently, understand the benefits of mathematical processing, and cultivate a positive attitude toward the voluntary use of mathematics in everyday life.</p>	<p>Interest/eagerness/attitude toward mathematics Mathematical thinking Expression/processing of numbers and figures Knowledge/understanding of numbers and figures</p>
2000s	<p>Zest for life Curriculum demonstrates greater selectivity of mathematics content and emphasizes mathematical activities of students while expanding the elective learning structure.</p>	<p>Impart fundamental knowledge and skills pertaining to numbers and figures through mathematical activities, cultivate skills that will allow students to have an outlook on everyday events and to think coherently, impart the fun and benefits of mathematical processing, and cultivate a positive attitude toward the voluntary use of mathematics in everyday life.</p>	<p>Interest/eagerness/attitude toward mathematics Mathematical thinking Expression/processing of numbers and figures Knowledge/understanding of numbers and figures</p>

Section 4.3: How are Curriculum Standards Improved and Implemented?

Yutaka Ohara

In Japan, schools develop their own education curricula based on standards established by the Ministry of Education, Culture, Sports, Science and Technology. Lesson Study is conducted either to make suggestions for the implementation of the curriculum, or to revise the curriculum.

1. Improving Curriculum Standards

Curriculum standards are comprised of the National Course of Study, which establishes the how schools are to develop their curricula, and the goals and content to be covered in each subject and grade level, and the School Education Law Enforcement Regulation, which establishes the number of required class hours. Curriculum standards are revised in approximately ten year cycles, and a transition period of three years has been allotted from the time any improvements are announced to the time of their full implementation. Revisions to the National Course of Study and School Education Law Enforcement Regulation are made in accordance with official procedures. First a report is compiled by a committee comprised of designated experts (Central Council for Education) that confirm the basic principles of the improvements that are to be made. This is followed by a report by a committee that confirms the revision guidelines and the class hours. Finally, a committee is convened to confirms the subject matter and teaching systems for each subject. Up until the most recent revisions, follow-up committees were going to be set up based on the results of the deliberations of these upper level committees, but in 2004 a

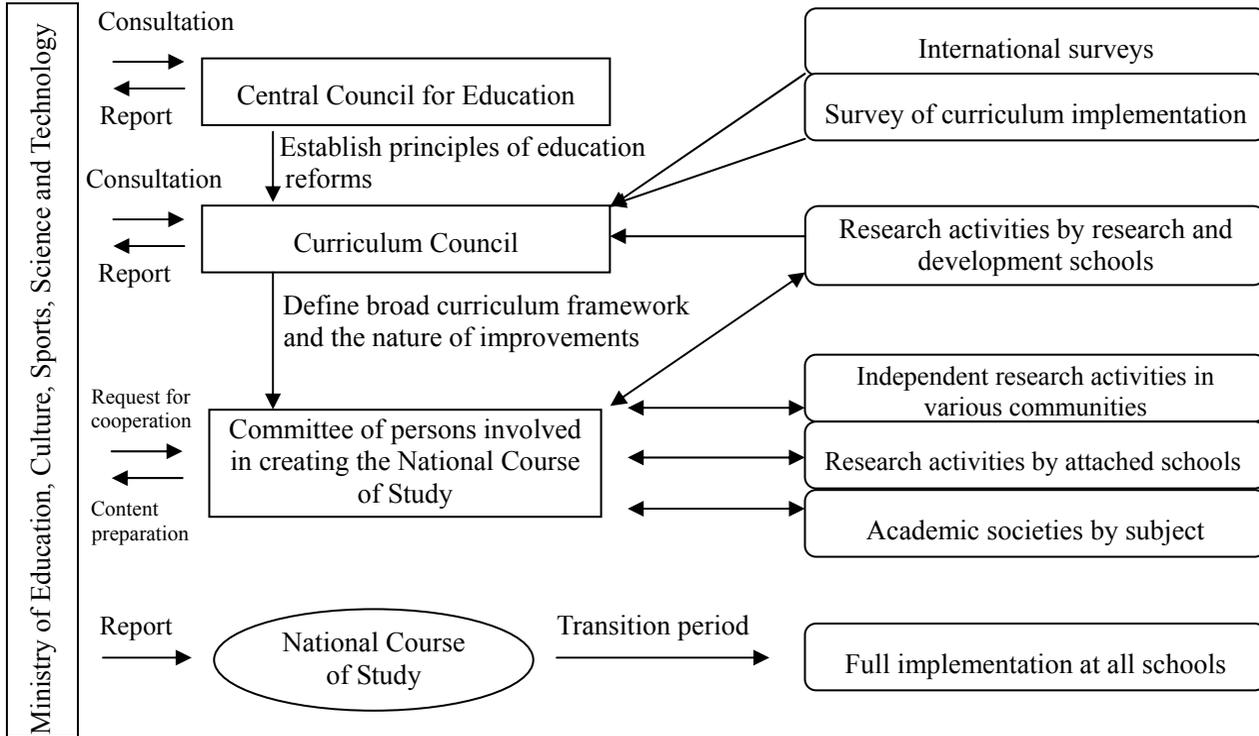


Figure 1: Process leading up to revisions

new format was adopted. Now a committee has been permanently established to fulfill the roles by these committees which will proceed with revision preparations concurrently.

The process of summarizing the objectives and subject matter to be covered in each subject, revising curriculum standards, and disseminating new information is overseen by MEXT's school inspector, school subject investigator, and other cooperative people. MEXT and the National Institute for Educational Policy Research (NIER) Curriculum Research Center perform various tasks related to implementing the curriculum in each subject and evaluating the status of implementation. For example, they conduct curriculum implementation status surveys during the transition period and after improvements have been made, issue recommendations for improving teaching methods to meet curriculum standards, and prepare revisions to the curriculum standards themselves. The school subject investigator promotes progressive Lesson Study on the revision and implementation of standards at designated research schools, and promotes improvements to standards based on proposals made by academic societies and boards of education.

2. Curriculum System at Each School

No matter how good a curriculum is developed, it has little meaning unless it is actually implemented. There are three types of educational curricula: the intended curriculum, the implemented curriculum, and the achieved curriculum. Policies need to be enacted to ensure that the intended curriculum is implemented and that the aim of that curriculum, student growth, is achieved. As a basis for implementing these policies, Japan has a system of educational laws governing these three types of curricula.

Reference materials to be used by individual schools when developing their own evaluation standards and methods.

(1) Explanation of National Curriculum Guidelines (MEXT)

A guide provided to explain and disseminate the key points and nature of revisions to the curriculum guidelines.

(2) Materials Pertaining to Individual-Oriented Instruction (MEXT)

Reference materials for teachers on each school's efforts to improve teaching strategies for promoting developmental learning and supplemental learning.

(3) Report on the Survey of Curriculum Implementation Status (NIER Curriculum Research Center)

A survey report that helps improve teaching by facilitating an understanding of how lessons based on the objectives and subject matter of each subject are implemented in the context of school curricula developed in accordance with the National Curriculum Guidelines.

(4) Reference Materials for Creating Evaluation Standards and Improving Evaluation Methods (NIER Curriculum Research Center)

Figure 2: Instructional materials issued by the national government to ensure implementation of the curriculum standards (sample)

The intended curriculum is developed by the school principal in accordance with the curriculum standards. The implemented curriculum is controlled by teachers who are responsible for using textbooks developed in accordance with the curriculum standards in their classes. The achieved curriculum is monitored by the use of student report cards that record the students' results for the year.

School education in Japan utilizes a textbook investigation system in which items that meet the curriculum standards are adopted for use as textbooks. For compulsory levels of education, that is, elementary and junior high school, textbooks are distributed free of charge. Because of financial restrictions, the number of pages and colors that can be used in these textbooks are limited. Recently there has been a trend toward relaxing those restrictions. If we want to achieve a distinctive education and promote developmental learning, textbooks need to be more individualized. At most until 10% of the content of textbooks for compulsory grade levels¹ and 20% of textbooks for upper secondary schools should contain content that goes beyond the scope of the curriculum standards. Textbooks are typically revised three times while a single set of National Course of Study is in effect.

Under the current curriculum standards, the evaluation standards used for filling out the Permanent Cumulative Record have shifted from relative to absolute standards. It has to be kept at least five years but it is kept permanently. The NIER has proposed a set of evaluation standards that will ensure that the curriculum standards are being met in teaching.

¹ Editors' note: A current topic of Lesson Study in compulsory levels is *the developmental and supplemental learning* to integrate additional contents on the textbooks.

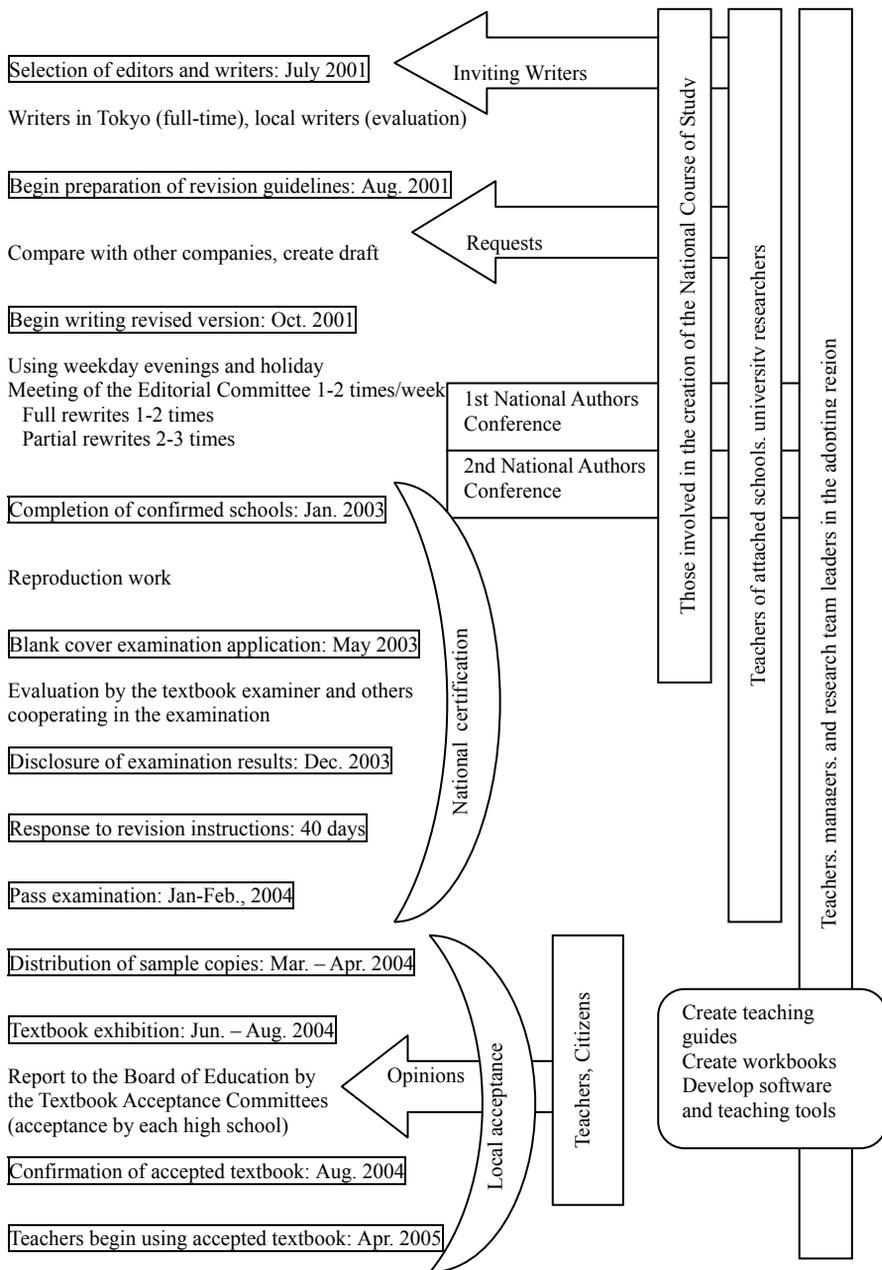


Figure 3: Sample process by which a textbook is revised and accepted

Section 4.4: How is Each School's Mathematics Curriculum Formulated and Implemented?

Shigeo Yoshikawa

The formulation and implementation of the curriculum is conducted internally within each school. Improvements are then made using feedback provided through the lesson study process.

1. Formulation of the School Curriculum

The school principal is responsible for developing the school's curriculum based on the National Course of Study, which embodies the curriculum standards, and the concerning laws and regulations. Each school creates an annual lesson plan and class schedule at the beginning of the school year based on the principal's instructions. The principal usually delegates the formulation activities to an internal department, like the educational affairs or research department of the school. The coordinating department oversees the annual teaching plan and class schedule created by the class teachers, teachers in charge of respective subjects, and the head teacher of each grade level. These plans are deliberated by the teachers' council, and the deliberation results are used by the principal to complete the formulation of the curriculum.

The general provisions of the National Course of Study confirm that the subject matter described in the plans are to meet certain minimum standards and require that each school formulate its own distinctive curriculum. To promote the development of a distinctive school and ensure that teachers are organized into independent and collaborative teaching groups, the principal prepares to put the right people in the right jobs starting in the previous school year and creates a system that facilitates his or her own ability to lead. As part of these preparations, the principal organizes internal departments like the educational affairs department or research department at the end of the previous school year. These departments cooperate on developing the school's unique

Evaluation Standards of the NIER Curriculum Research Center

Table 1: Internal Organization in a School

Department/coordinating body	1st Year Team	2nd Year Team	3rd Year Team	...
Educational Affairs Department: Planning and implementation of the curriculum	Teacher A (grade level officer, Japanese subject)	Teacher C (mathematics coordinator)	Teacher E (grade level officer)	
Research Department: Planning and implementation of teacher training	Teacher B (mathematics research leader)	Teacher D (grade level officer, Japanese subject)	Teacher F, mathematics research leader	
...				

Principal = vice principal = educational affairs head teacher (other than the class teacher)
 = Teachers meeting (all teachers, called by the principal)
 = Department meetings (held by the head teacher of each department)
 = Grade level meetings (held by head teacher of each grade level)
 = Subject meetings (held by head teachers of each subject)

Announced to each school level (the following shows the case of an elementary school)

Chapter 1 General provisions (explanation of how to formulate a curriculum for that school level)

Chapter 2 Explanation of the subject matter to be taught in each subject

Section 3 Mathematics 3.1 Objectives

Impart fundamental knowledge and skills pertaining to numbers and figures through mathematical activities, cultivate skills that will allow students to have an outlook on everyday events and to think coherently, impart the fun and benefits of mathematical processing, and cultivate a positive attitude toward the continued use of mathematics in everyday life.

3.2 Goals and subject matter of each grade level

1st Year: 1. Objectives (explains the goals of each grade level)

1.1 Impart a strong sense of numbers through activities that make use of actual objects. (Remainder omitted.)

2. Subject matter (explains the subject matter for the grade level in each subject)

A Numbers and Mathematics 1. Understand the meaning of and be able to use numbers through activities like counting the number of objects.

a. Compare the number of objects by performing correspondence operations. (Remainder omitted.)

Terms, symbols 3. Handling of the subject matter (each grade level)

3.3 Creation of teaching plans and the handling of subject matter across grade levels

Figure 1: Composition of the National Course of Study

curriculum (the educational goals of that particular school) and issues proposals to the principal and teachers. Through a meeting of the teachers' council, the principal asks each teacher to formulate a curriculum that achieves the distinctive goals of the school. If the principal follows the procedures for formulating the school's curriculum under the agreement of the teachers as a group, the curriculum for each class becomes curriculums that are determined by individual teachers. The implementation of the curriculum is the individual responsibility of each teacher as well as the collective responsibility of the teachers of the school as a group.

2. Preparation of an Annual Teaching Plan

The mathematics curriculum is created and implemented by the class teachers as their annual teaching plan. The National Course of Study, which stipulates the legally required subject matter, only provides basic guidelines regarding the objectives and subject matter to be covered in each grade level and the preparation of teaching plans. MEXT issues manuals and instructional materials to help teachers better understand the key points of those guidelines. Because annual teaching plans are created by the school, the National Course of Study outlines the subject matters to be taught in each grade level, but do not designate the order in which that information is to be taught. To respect the creation of a distinctive curriculum at each school, the subject matter for each grade level should be systematically planned in advance, within a range that is not too difficult for students.

Annual teaching plans stipulate the subject matter to be covered for the year and the teaching objectives. Teachers in charge of each class plan the school's objectives (the distinctive goals mentioned above) based on the National Course of Study, and then plan the subject matter to be taught, the order in which to teach it, and the objectives for the specific teaching term based on the goals of the teachers they oversee. This is an activity that relies heavily on the teacher's wishes and capabilities. MEXT and NIER publish materials and evaluation regulations for supporting the creation of

Table 2: Four Evaluation Perspectives in the form of statements that specify mathematics objectives

Interest, eagerness, attitude toward mathematics	Mathematical ways of thinking	Expression and processing of numbers and figures	Knowledge and understanding of numbers and figures
Has an interest in mathematical phenomena, appreciates the fun of the activities and the benefits of the mathematical process, seems willing to try to apply what he/she has learned to everyday phenomena.	Has acquired the fundamental mathematical ways of thinking through mathematical activities, establishes a plan of action with an outlook for what lies ahead.	Has acquired the skills involved in expressing and processing numbers and figures.	Has a strong sense of numbers and figures and understands their meaning and properties.

(1) A. Numbers and Calculations

Content of the National Course of Study

- 1 Understand the meaning of and be able to use numbers through activities like counting the number of objects.
 - a. Compare the number of objects by performing correspondence operations.
 - b. Correctly count and express a number of items and identify their order.
 - c. Create a sequence and express it on a number line by thinking about the size of numbers and their sequence.
 - d. Look at a number as the sum or difference of other numbers, assign relationships between numbers.
 - e. Understand the meaning of numbers up to 100 and how to express them.
- 2 Understand the meaning of addition and subtraction, and know how to use them.
 - a. Know when addition and subtraction are used, express those calculations as formulas, and read those formulas.
 - b. Think about the process of performing addition or subtraction calculations of two single-digit numbers, learn to perform those calculations properly.
- 3 Collect, count, and divide specific objects equally, organize and express those objects.

Figure 2: Evaluation standards for each subject matter (1st year)

well developed teaching plans and provide support for these activities.

3. Promoting Lesson Study and Determining Results

At the beginning of each school year, in addition to its annual teaching plan, each school establishes action plans, teacher training plans, a class schedule for each grade level, and the schedule for teachers' organizational activities. The teachers' organizational activities include weekly subject meetings for each academic subject, grade level meetings for each grade level, and educational affairs department meetings for each department. Participants in these meetings discuss the details of the curriculum implementation and make revisions to established plans. Training sessions for improving lessons based on objectives and examining student learning are held about 10 times a year in the form of research lessons. A teaching supervisor from the board of education is invited to observe these lessons, and often provides suggestions about whether teaching is being conducted appropriately based on the curriculum standards.

The lesson study results of each school are shared with others through magazines for teachers, organizational research reports issued by the board of education, and research meetings held by academic societies. MEXT and the NIER Curriculum Research Center study teaching practices and student achievement through visits to research and development schools, seminars by teaching consultants, and curriculum implementation surveys. Using this information to obtain clues for making improvements, they develop policies to help improve lessons. This is done by using MEXT publications and teaching materials that summarize the curriculum standards, and by working with cooperative individuals.

Table 3: Evaluation Standards for "A: Numbers and Calculations "

<p>Enjoys working with numbers and calculations and strives to gain various experiences with them.</p>	<p>Through mathematical activities like measuring the actual size and order of objects, and expressing their size in a chart or formula, student devises creative strategies and develops ways for expressing numbers and performing mathematics calculations.</p>	<p>Can easily perform simple addition and subtraction problems using whole numbers.</p>	<p>Has a strong sense of numbers, knows how to express the meaning of numbers, understands the meaning of addition and subtraction calculations performed on whole numbers (natural number and zero).</p>
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Table 4: Sample evaluation standards for each subject matter (partial)

<p>Correctly counts and expresses the number of objects, such as concrete objects. Appreciates the benefits of using numbers to express the quantity or order of objects. Appreciates the benefit of being able to use numbers to know the size or sequence of numbers.</p>	<p>Through exercises involving numbers, knows how to read and express numbers and how to think about the size and sequence of numbers.</p>	<p>Can read and express numbers up to 100. Can correctly count and express the number of objects and identify their order.</p>	<p>Has a strong sense of the size and structure of numbers. Looks at a number as the sum or difference of other numbers, has a strong sense of the structure of numbers. Numbers up to 100.</p>
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Section 4.5: Teaching and Assessment Based on Teaching Guides

Masao Tachibana

1. The Purpose of Assessment

Assessments are educational activities conducted for the purpose of helping teachers improve their lessons and for the better growth of students.

Assessments are not conducted for the mere sake of assessing, but to enable students and teachers to guide their learning situation in a positive direction and to reflect on their learning and teaching.

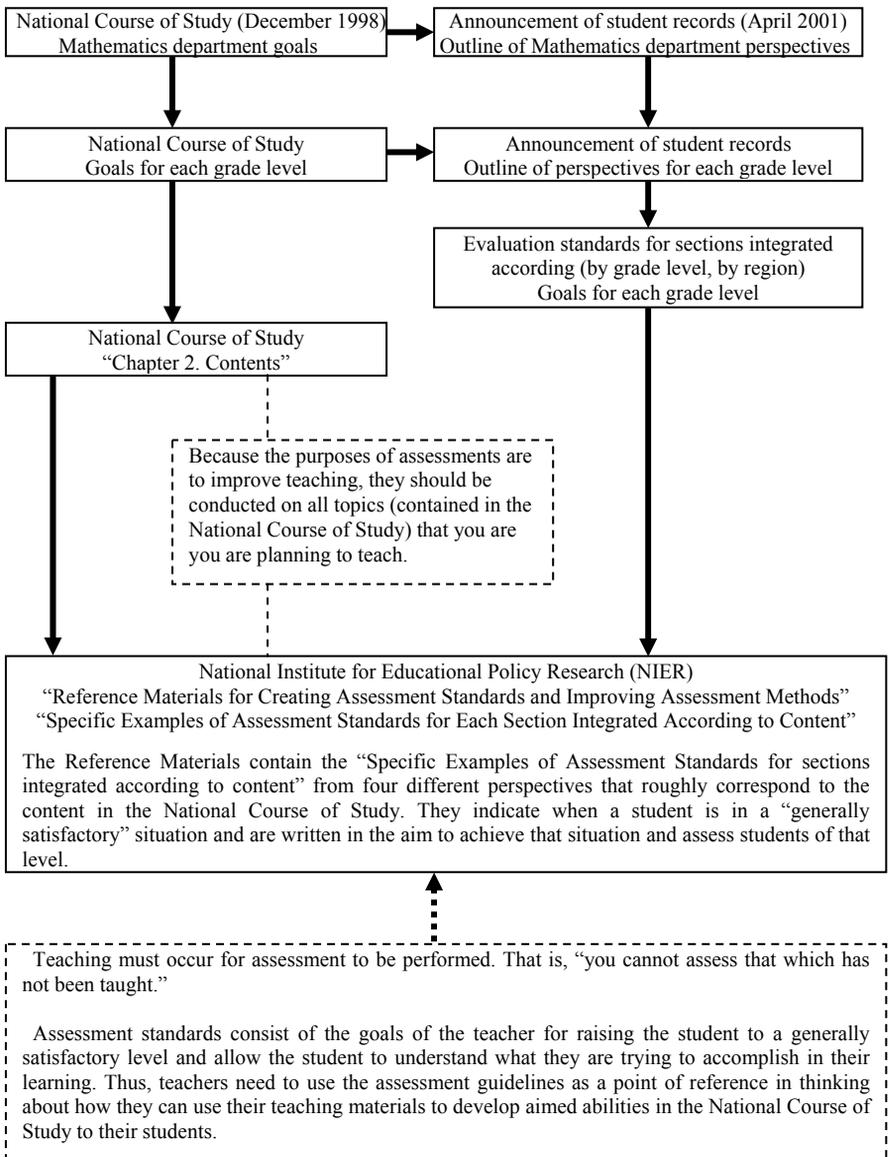
When students finish studying a particular topic, an assessment is carried out to see whether the goals of their activities were achieved. If the student's performance is not what was hoped, some teachers bemoan student's inattentiveness or feel disappointed in their students. This is not an appropriate response.

Since the primary goal of assessment is to ensure that teaching is well-suited to the students' learning capabilities, it is important that the teacher has a solid understanding of their students' learning status. Thus, activities to observe and measure students' capabilities through the use of tests and other means are often incorporated into assessments of students. Traditionally, assessments have tended to be thought of as a process of obtaining this kind of information and expressing it as a number or score. Given their objectives, however, assessments are significant in and of themselves as resources that can be used to help teachers and students examine and improve their own behavior.

2. Assessments that Improve the Teacher's Lessons

Conducting a lesson based on a goal makes it possible to evaluate the lesson, and through that evaluation, lessons will improve.

Relationship Among National Course of Study as Standards, Teaching Materials, and Journals



Once the study of a particular topic has ended, it is important to identify, based on the goals of the lesson, how effectively the students learned, what kinds of issues they had trouble with and which students struggled with what content.

By identifying these things, teachers can examine whether the learning activities and a teacher's method of teaching and supporting students are effective for the achievement of the curricular goals. Depending on the situation, it is possible to revise the teaching plan, improve what needs to be improved, and enable students to achieve the goals of the curriculum.

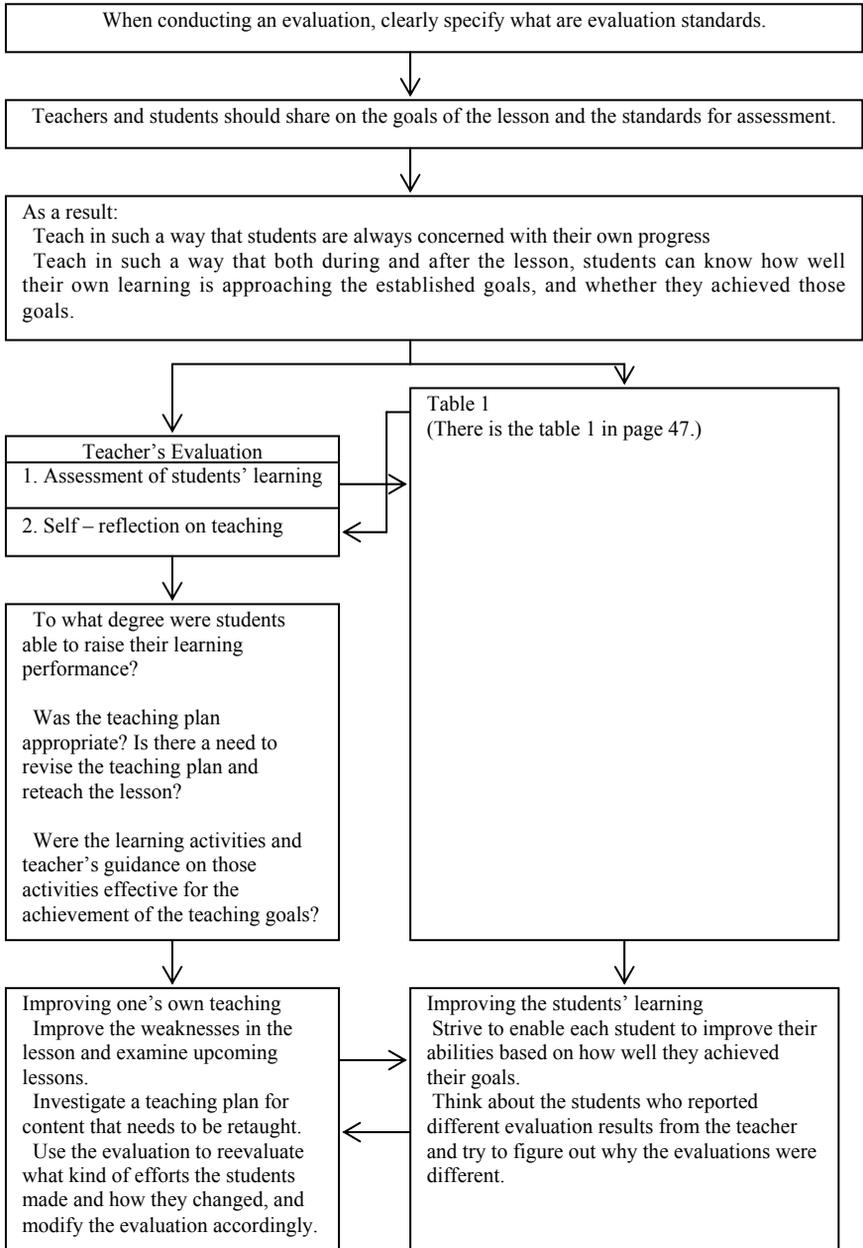
Assessments should reflect the Mathematics goals stipulated in the National Course of Study and the goals and curricular content for the particular grade level. The Reference Guides for Creating Evaluation Standards and Improving Evaluation Methods issued in February 2002 by The National Institute for Educational Policy Research (NIER) were created based on the National Course of Study and the descriptions and outline of the viewpoint of those guidelines. Thus, they can be used as a reference for teaching and conducting assessments.

3. Assessment to Improve Student Learning

Assessment should act as a “message from the teacher” to help improve the student’s academic ability.

When conducting an assessments of students’ work, it is important that the teacher conveys evaluative information that is specific and easy to understand, such as, “This is how much you were able to do,” “You started out a little slow in this area, but you worked hard and were able to improve,” or “You’ll do even better in the future if you pay particular attention to this.” In using assessment, teachers must be careful to avoid pointing out a student’s weaknesses or shortcomings, but encourage student learning by identifying areas where the student needs to expend more effort. Students who are not achieving their goals often do

Evaluation Standards for Teaching, Standards for Student Assessment from Four Perspectives to Reflect and Foster Student Growth



not know what they are supposed to be doing. Thus, when a teacher realizes that a student is not achieving his or her goals, it is important that he/she thoroughly investigates why this is happening, make the student aware of the problem, provide instruction so that the student can quickly fix the problem, and enable the student to engage in meaningful learning. The teacher must clearly indicate his/her intent to help the student overcome his or her difficulties. If the problem seems too difficult for the student, the teacher can make learning easier by breaking the problem up into small steps or starting with a more simplified version. Simply saying, “Do your best” may be encouraging, but it does not amount to teaching. When informing a student of the results of their assessment, it is important that the student walks away from the encounter feeling inspired to tackle and overcome the challenges.

Editor’s note: In Japanese, the same word “*Hiyoka*” is used for ‘assessment’ and ‘evaluation’ and enhanced the feedback. Where the author refers to measuring students’ achievement or otherwise evaluating their performance, it would appear to be better to use the word ‘assessment’. But in cases where the results of assessments are used by teachers to appraise the effectiveness of their teaching, the word ‘evaluation’ seems more appropriate. However, in the case of translated titles of official documents, such as those cited above, which use the term ‘evaluation’, it is not appropriate to change their wording. Readers should understand that the focus of these documents is on appraising student performance and enhancing achievement. As the author suggests, many of these assessments are formative in nature, being intended to provide information to students to improve their learning and to provide feedback to teachers on effectiveness of their teaching.

Table 1

		Students' Self – Assessment	
		Achieved goal	Did not achieve goal
Students Evaluation by Teacher	Achieved goal	Tell student, “If you pay attention to these particular things, together we'll be able to work to even further improve your performance.”	Some students may be too hard on themselves, so instruct them to confirm the goals of the lesson and have confidence as they work through their lessons.
	Did not achieve goal	Some students may be too easy on themselves, so reconfirm the evaluation standards and instruct the attitude toward learning that is to be achieved.	Determine and make the student aware of the reasons they did not achieve their goals, provide advice to help them improve their performance quickly, and provide guidance to help the student engage in meaningful learning.

Section 4.6: Textbooks and Teaching Guides

Takeshi Miyakawa

Under the Japanese school system, teachers must use textbooks that have been approved by the Ministry of Education, Culture, Sports, Science and Technology. There are six types of mathematics textbooks for elementary and junior high schools and about twenty types, by grade level, for high school. These are published by private publishing companies. The elementary school mathematics textbook, which excludes first grade mathematics, consists of about 100 B5-sized pages divided into two volumes. The junior and senior high school textbooks consist of 100 to 200 A5- or B6-sized pages. In compulsory education, elementary and junior high school level, textbooks for students are distributed for free by the national government. The numbers of pages were restricted by the government and then, they have been compiled to avoid any overlap in the curriculum across different grade levels and to ensure that students can learn all of the necessary content, and complete their practice exercises, in the number of hours allotted for the school year.

Textbook publishing companies also publish accompanying texts for teachers called teaching guides. Their format and structure varies somewhat by publisher, but they are usually comprised of a *practical* handbook, which explains textbook articles in red and teaching development methods, and a *theoretical* handbook of about the same size.

The practical handbook contains information on teaching systems, goals for each unit, and teaching plans, and provides highly detailed information about the teaching process, such as questions to ask students and their probable responses, and other teaching essentials.

4月10日

第1時 1 対称図形 (9時間)
2 多角形と対称 (3時間)

準備
(教) 四角形の切り抜き
(児) 表, 四角形のプリント

① 教科書の4つの四角形を提示する。

② 線対称な図形について調べる。
・線対称な図形はどれか考えさせる。
・線対称な図形で、対称の軸は何本あるか調べる。
・図の中に、対称の軸をかきこませる。

③ 点対称な図形について調べる。
・点対称な図形はどれか考えさせる。
・点対称な図形で、対称の中心を調べさせる。
・対応する点を結んで対称の中心を見つける。

④ 調べたことを表にまとめさせる。
・教科書の表について、平行四辺形の場合を説明する。
平行四辺形は、線対称でないから、×と書いてある。対称の軸はないからとなる。
点対称な図形が○とつく。
・平行四辺形の場合に合わせて、表を完成する。

⑤ 対称の軸が1本だけの四角形をかく。
・いろいろ考えさせる。
・発表させる。確かめる。
・点対称になるか調べさせる。

⑥の例

等脚台形 たこ形

4月10日

第1時のねらい
・対称性に着目して四角形を考察し、対称の軸の数、対称の中心を見つけることができる。(④、⑤)

3 多角形と対称
① 四角形の線対称、点対称の観点から分類する

(平行四辺形) (長方形) (ひし形) (正方形)

上の四角形について、次のことを調べましょう。

② 線対称な四角形はどれでしょう。また、線対称のとき、対称の軸は何本あるでしょう。

③ 点対称な四角形はどれでしょう。また、点対称のとき、対称の中心はどこでしょう。

●調べたことを表にまとめましょう。

	線対称	軸の数	点対称
平行四辺形	×	0	○
長方形	○	2	○
ひし形	○	2	○
正方形	○	4	○

② 対称の軸が1本だけある四角形をかいてみましょう。(定規使用)
その四角形は、点対称な図形になるでしょう。(定規使用)

指導のポイント
本時は、線対称、点対称の観点から基本図形を見なおすことが指導のポイントである。既習事項では、辺の平行、角、辺の長さ、対角線などをもとに基本図形を弁別したり、自然図形(無数の曲線は付いていない)などをも考察してきた。本時の学習で図形の見方が広がるということが限りつつある。基本図形では、線対称の図形の場合、対称の軸は1本とは限りないことに注意させる必要がある。線対称であるかどうかの判別は、1本の対称の軸があれば十分である。切り抜いた図形を使って、いろいろに折ってみることが理解を確かめなにもするのである。

ねらい
① 対称性に着目して多角形を考察することができる。(④～⑤)

3 多角形の線対称、点対称からの考察
③ ① 上三角形、二等辺三角形、正三角形
② 線対称な三角形はどれでしょう。また、線対称のとき、対称の軸は何本あるでしょう。正三角形、3本
④ 点対称になっている三角形はあるでしょうか。(ない)

④ 正多角形について、それぞれ線対称な図形か、点対称な図形かを調べて、下の表にまとめましょう。

	線対称	軸の数	点対称
正三角形	○	3	×
正四角形(正方形)	○	4	○
正五角形	○	5	×
正六角形	○	6	○

⑤ 円についても、線対称な図形か、点対称な図形かを調べてみましょう。
線対称な図形で、対称の軸は無数にある。
点対称な図形でもあり、対称の中心は円の中心になる。

4月10日

第2時 1 対称図形 (9時間)
2 多角形と対称 (3時間)

準備
(教) 三角形、正多角形の切り抜き
(児) 正多角形のプリント

① 教科書の3角形を提示し、線対称な三角形について調べさせる。
・必要な場合、いろいろな三角形をかかせ、切り抜かせるように。
・折って重ね、対称の軸をみつけれさせる。
・線対称な三角形について、対称の軸は何本あるか調べさせる。

② 点対称になる三角形について考えさせる。
・いろいろな三角形をかかせ、点対称な三角形になるか調べさせる。

③ 正多角形の図を示し、線対称、点対称について調べさせる。
・ワークプリントで対称の軸の数、対称の中心を調べさせる。
・表に整理させる。
・表で気がついたことを発表させる。
正多角形で、辺の数が偶数のときは点対称ではない。
正多角形はすべて線対称である。
対称の軸の数は角(辺)の数と同じ
……など

④ 円について線対称な図形か、点対称な図形かを調べさせる。
・円をかかせ、自由に調べさせる。
・線対称でも点対称でもある図形である。
・対称の軸の数は無数にある。

Figure 1: Pages from a sixth grade mathematics teaching guide published by Keirinkan. It provides explanations and answers to the textbook pages shown in a rubric in the frame in the middle of the pages, and shows the lesson sequence, teaching tips, and supplementary questions around the periphery.

The problem-solving approach characteristic of Japanese classes is derived from Lesson Study based on both theoretical and practical components. Figure 1 shows pages from an actual teaching guide *practical* handbook.

Lesson Study begins with materials research, and the meanings and significance of the specialized terminology and teaching materials used in mathematics education, which are necessary for this process, are explained in the *theoretical* handbook. This specialized terminology consists of terms specific to the field of mathematical education, as distinct from specialized mathematical terms or pedagogical terms. Take, for example, the terms “partitive division” and “measurement division,” which are used to classify types of division problems (see Figure 2).

Editors' Note: Workbooks and Journals

Teachers usually use a workbook of exercises for students additionally.

There are a number of teachers' journals of mathematical education. They may be categorized in two. First type of journals is aimed to share the way of interesting teaching including subject matter and worksheet for children in a classroom. Second type of journals is focused in more developmental and challenging research with the terminology in teacher's guides and includes the reconstructed protocol of teaching process with major teacher's questioning, children's multiple answers and discussion.

Those Workbooks and Journals are used for sharing the experience of Lesson Study as well as teachers' guides.

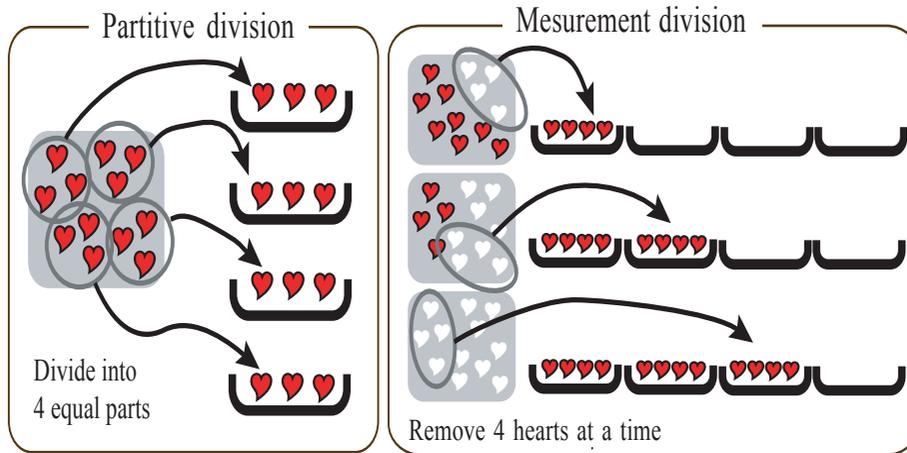


Figure 2: This image explains the terms “partitive division” and “measurement division” using the division problem $12 \div 4$. “Partitive division” refers to dividing a whole into several equal parts, while “measurement division” refers to dividing a whole into groups of a certain number of elements.

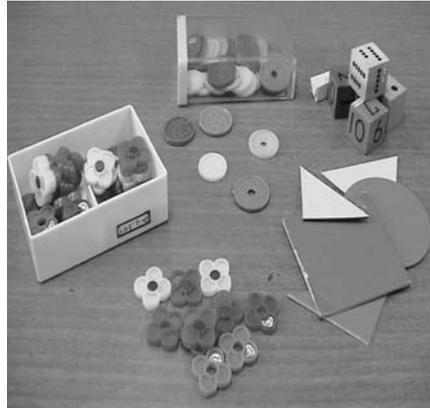
Section 4.7: What Kinds of Teaching Materials and Aids are Used in Japan?

Hiroko Tsuji

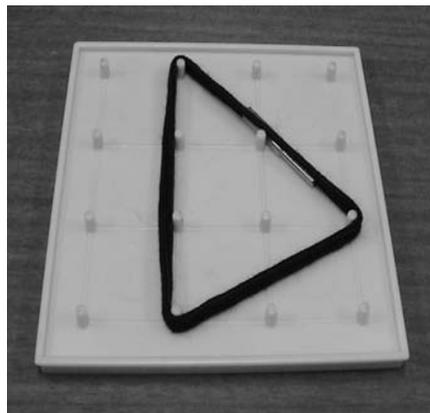
Japanese classrooms utilize creative teaching materials and aids to enable students to experience the benefits and fun of mathematical ways of thinking and to cultivate a strong sense of quantities and figures in students.

In mathematics classes in the lower elementary school years, each child has their own mathematics activity set which the teacher uses regularly in the everyday course of teaching (Figure 1). Students use aids like geoboards and pattern blocks in elementary school classrooms, and teachers develop creative lessons that utilize these items. These aids are systematically chosen and used based on the nature of the activity.

Because of the need for ICT education, however, the national government is promoting the digitization of textbooks, teaching materials, and teaching aids by the textbook companies. Universities and boards of education promote the development of collection of links that support children's learning, and the national government has established the National Information Center for Educational Resources (Figure 2). This information is used in individual classrooms through video presentations. While traditional calculation devices, such as the abacus, are taught, students are also learning to use calculators and computers (Figure 3). Some schools are even promoting cooperative learning through groupware.



Mathematics activity sets



Geoboard
Figure 1

Rather than just looking at teaching aids as tools for imparting knowledge, teachers have to develop mathematics lessons designed to create a certain type of learning environment. They need to cultivate an environment that encourages students to learn and think on their own, and to pursue their own interests and their desire to know why and how things work.

In Japanese schools, students use workbooks in addition to textbooks. Workbooks are often used for the work students do outside of class hours. They help students absorb mathematical ways of thinking and expression, which they obtain through operational and experiential activities, and develop them into knowledge and skills.

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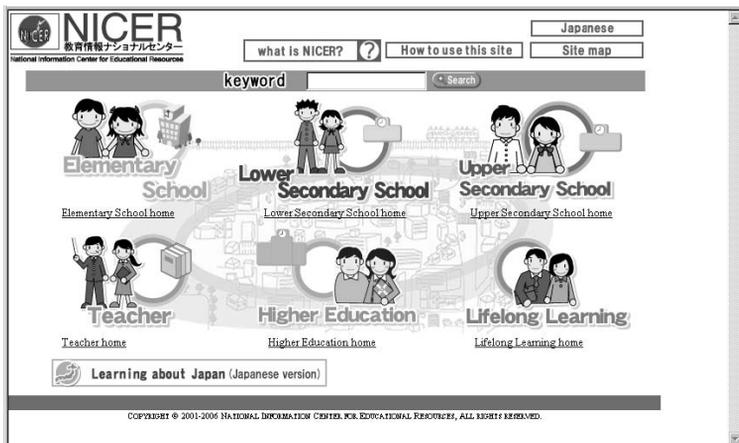
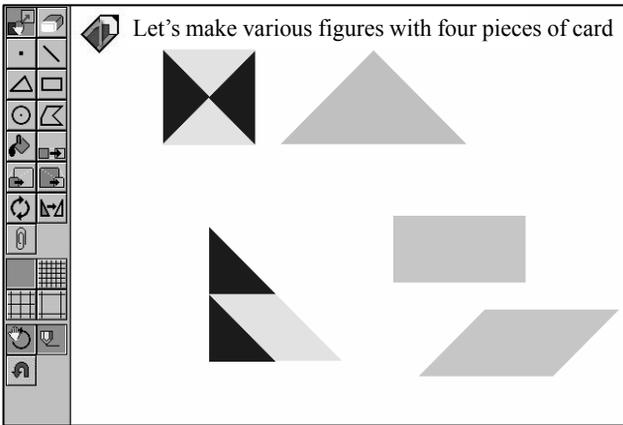
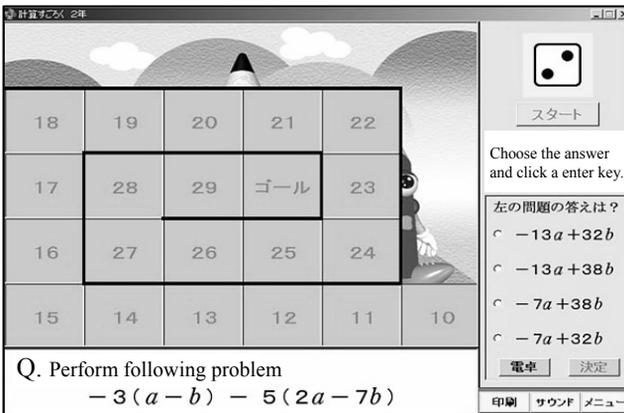


Figure 2



Dainippon Tosho: Educational Mathematics software



Keirinkan: Junior high school educational Mathematics software, Masunabi Do!

Figure 3: Sample efforts of the textbook companies

Section 4.8: What do Teachers and Teacher Trainees Think About Lesson Study?

Tadayuki Kishimoto

1. Teacher Trainee Attitudes toward Lesson Study

Under Japan's teacher training program in four year college (Table 1), prospective teachers must participate in a four-week period of teaching practice to become licensed to teach at the elementary and junior high school levels, and a two-week period of teaching practice to become licensed at the senior high school level. In addition to the actual practicum, they must receive preparatory instructions before the practicum and follow-up instructions afterwards.

Table 2 shows the results of a survey conducted by Nukui and Hirose (1997). This study examined the changes in attitudes held by 64 teacher trainees before and after they participated in their practicum. Before the practicum, teacher trainees reported a desire to have a classroom in which they would respect the independence of their students and prepare learning materials well suited to the actual needs of the students.

After the practicum, however, the teacher trainees became aware of the differences in the lesson plans they had prepared and the way the actual classroom experience unfolded, and they experienced difficulties in trying to communicate with their students.

Table 1: Necessary Credits of teacher’s certification for bachelor degree in undergraduate program.

	Elementary	Middle	High
A) Academic Subjects	8	20	20
B) Subjects for Teachers	41	31	26
C) Subjects of A or B	10	8	16
D) Other Subjects	8	8	8

A) Special Academic Area. B) Including teaching methodology. D) Including Study of the Constitution

Table 2: Changes in the Attitudes of Teacher Trainees Before and After a Practicum (Nakai and Hirose, 1997)

Attitudes Before the Period of Teaching Practice
<ul style="list-style-type: none"> • I want a classroom where students can freely express their opinions. • I want a classroom where the teaching materials are prepared based on an understanding of the students’ needs. • I want a classroom where children can learn independently. • I want to incorporate students’ ideas in the course of class. • I want a classroom in which people can listen carefully to one another.
Attitudes After the Period of Teaching Practice
<ul style="list-style-type: none"> • I was able to develop the lesson while having fun communicating with the students. • I tried to understand the individual personality of each student based on their everyday behaviors, and tried to communicate with them. • I didn’t know what to do when the children reacted differently than I’d expected. • I realized that there are a lot of gaps between what the students want and what teachers want. • There were major differences between what I planned and what actually happened in the classroom. • Even if I understand a subject, it is difficult to study it along with the students and make them understand.

2. Teacher Attitudes toward Lesson Study

Table 3 shows the results of a survey conducted by Elementary School Section of the Research Department (2001) of Japan Society of Mathematical Education among 476 elementary school teachers in 2000.

According to this study, Japanese math teachers tend to:

- (1) strive to facilitate communication between themselves and their students,
 - (2) incorporate specific hands-on and experience-based activities, such as experimental measurements,
 - (3) try to improve students' problem-solving capabilities by presenting problems that can be solved in a variety of ways,
 - (4) strive to impart theoretical understanding while simultaneously conducting skills practice,
- and
- (5) rarely conduct classes using an explanation-driven model in which the teacher gives an explanation and the students then solve problems.

Table 3: Teacher Attitudes Toward Lesson Study
(Elementary School Section: Research Department, 2001)

Topic	Always	Frequently	Occasionally	Never
My class emphasizes problem-solving according to the pattern: problem presentation → independent work → development → summarization	11.9%	47.2%	37.1%	3.9%
I use problems that have multiple correct solutions.	1.1%	21.2%	52.6%	25.1%
I focus on having the students create problems rather than solving problems.	0.2%	11.5%	75.9%	12.4%
I respect the students' ideas and develop lessons based on my interactions with them.	17.3%	48.9%	32.0%	1.8%
I take students outside the classroom to gather materials or take measurements.	4.7%	32.4%	57.5%	5.4%
I emphasize tasks or creative activities that have mathematical content.	1.5%	25.4%	66.5%	6.6%
I use a textbook and plan lessons based on its lesson progression.	16.9%	40.1%	38.6%	4.4%
My class emphasizes skills practice and emphasizes the completion of workbook lessons.	4.2%	33.5%	55.3%	7.0%
I start out by explaining the important points and then later have students solve problems.	5.8%	28.4%	44.8%	21.0%

Comparisons of Features of Past International Comparative Studies

Section 5.1: Why Have Japanese Lessons Paid Attracted Attention and What Are its Features?

Hanako Senuma

1. Improving the Curriculum and Teaching Methods to Improve Academic Abilities

Since the Second International Mathematics Study (SIMS) was conducted in 1981 (Table 1), the US has been implementing curriculum revisions for the purpose of raising the level of academic abilities. Japan was the top of 20 countries in the SIMS seventh grade. The main reason that Japan obtained such a high score was accredited to the high level of its mathematics curriculum. But it is the teachers who are actually responsible for improving the curriculum. Thus, a video study on “A Comparison of Mathematics Instruction in Germany, Japan, and the US” was conducted on eighth grade as an optional study on the 1995 Third International Mathematics and Science Study (TIMSS 1995). This showed that Japanese mathematics lessons emphasize the process of problem solving, and since then several other countries have been hoping to improve academic abilities by conducting Japanese style lessons. However, it has already been 10 years since those classroom videos were taken, and today in Japan many new teaching methods, such as separating classes by level of proficiency, are being implemented.

Table 1: A focus on Japanese lessons by comparison of international study

International Association for the Evaluation of Educational Achievement (IEA) Main Study	TIMSS Video Study (IEA Option)	Year	Major factors that attention paid to Japan
Second International Mathematics Study (SIMS)		1981	
		1983	<i>A Nation at Risk</i> (Points out the low level of mathematics and science skills in the USA) Japan was the top of 20 countries in the eighth grade
		1986	Japan - USA seminar on Mathematical Problem Solving (A comparison of problem solving in the USA and Japan becomes the opportunity to introduce Japanese lesson in the USA. <i>The Open – Ended Approach</i> , a book on Japanese teaching methods, is translated into English in 1993.)
		1987	<i>The Underachieving Curriculum</i> (USA). (Argues that Japan’s high scores are due to excellence of curriculum.)
		1989	<i>Curriculum and Evaluation Standards for School Mathematics</i> (USA). (Suggests that establishing a common curriculum nationwide will improve mathematics skills.)
		1991	<i>Professional Standards for Teaching Mathematics</i> (USA) (Training of high quality teachers is important for raising academic skills.)
Third International Mathematics and Science Study (TIMSS 1995)	Comparison of mathematics lessons in three countries	1995	
Third International Mathematics and Science Study – Report (TIMSS 1999)	Comparison of mathematics lessons in seven countries	1999	Results from mathematics lessons in three countries (TIMSS 1995 video study) increased interest for Japanese Lesson Study. (Japanese lessons focus on problem solving, which is linked to higher scores.)
<i>Trends</i> International Mathematics and Science Study (TIMSS 2003)		2003	Results are released on mathematics lessons in seven countries. (Show that Hong Kong and Japan, are both high scored countries and employ different teaching styles.)

2. Pose Problems to Raise the Level of Mathematical Ideas and Switching Lessons Between Whole Classroom Work and Individual Work

In 1999, eighth grade lessons in seven countries were videotaped as an optional study component of the TIMSS 1999 and the results were released in 2003. They indicated that Japanese teachers control their lessons well; the goals and summary statements are presented, teachers pose problems that require students to think (Figure 1), problems are presented that improve the students' abilities to make connections between ideas (Figure 2), alternative methods of solution for problem are examined, and teachers switch between whole classroom work and individual work as appropriate. The report showed that Hong Kong SAR and Japan had similarly high scores, but that they employ different teaching methods, thereby indicating that there is no single teaching method for improving academic abilities.

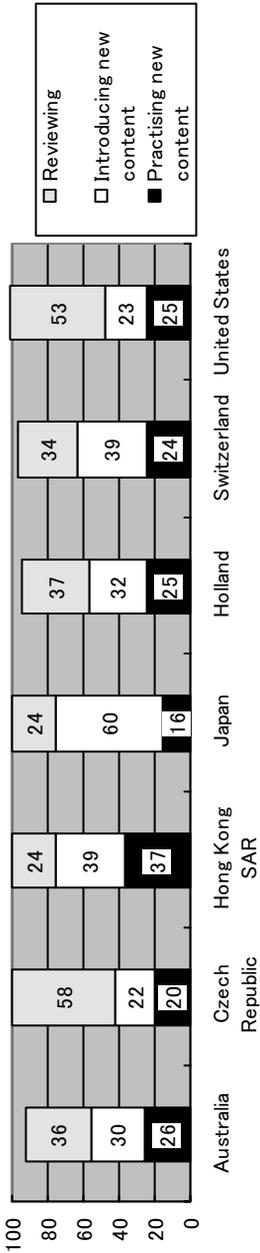


Figure 1: Japanese lessons emphasized introducing of new content

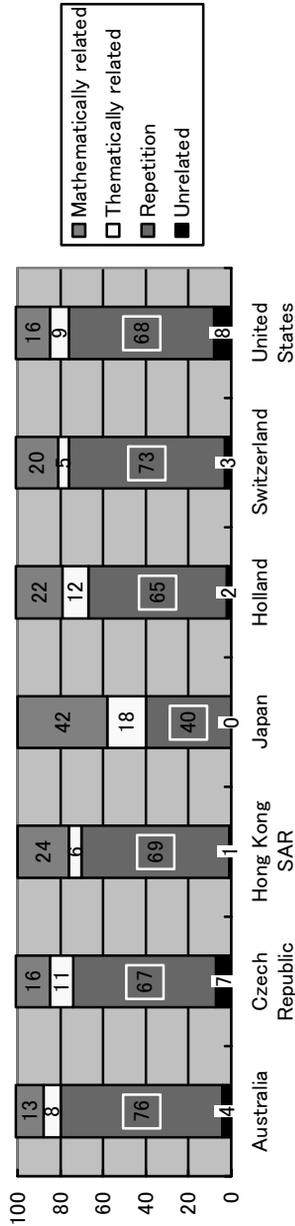


Figure 2: Japanese lessons designed to gradually raise the level of mathematical ideas

Recommended Websites: http://www.mext.go.jp/a_menu/shotou/shingi/index.htm, <http://nces.ed.gov/timss/video.asp>, <http://timss.bc.edu/>

Understanding Japanese Mathematics Lessons

Section 6.1: How do Japanese Teachers Explain and Structuralize Their Lessons?

Yoshinori Shimizu

Japanese lessons as “structured problem solving”

The following sequence of five activities has been described as the Japanese lesson pattern: reviewing the previous lesson; presenting the problems for the day; students working individually or in groups; discussing solution methods; and, highlighting and summarizing the main point (Table 1).

Table 1: The Japanese Lesson Pattern (Stigler & Hiebert, 1999, pp.79-80)

Reviewing the previous lesson
Presenting the problems for the day
Students working individually or in groups
Discussing solution methods
Highlighting and summarizing the main point

Teacher's Roles During the Lessons: Some Pedagogical Terms Shared by Teachers

Japanese teachers play several roles at each stage of their lessons. The following pedagogical terms are commonly used to describe such teachers' key roles.

"Hatsumon" at the presentation of a problem

"Hatsumon" means asking a key question for provoking students' thinking at a particular point in a lesson. At the beginning of the lesson, the teacher may ask a question for probing or promoting students' understanding of the problem. In a whole-class discussion, on the other hand, he or she may ask, for example, about the connections among the proposed approaches to the problem or the efficiency and applicability of each approach.

"Kikan-shido" during problem solving by students

"Kikan-shido", which means an "instruction at students' desk", includes a purposeful scanning by the teacher of students' problem solving on their own. The teacher moves about the classroom, monitoring students' activities mostly silently, doing two important activities that are closely tied to the whole-class discussion that will follow. First, he or she assesses the progress of students' problem solving. In some cases, the teacher suggests a direction for students to follow or gives hints to the students for approaching the problem. Second, he or she will make a mental note of several students who made the expected approaches or other important approaches to the problem. They will be asked to present their solutions later. Thus, in this period of the purposeful scanning, the teacher consider questions like "Which solution methods should I have students present first?", or "How can I direct the discussion towards an integration of students' ideas?" Some of the answers to such questions are prepared in the planning phase but some are not.

"Neriage" in a whole-class discussion

There is a term for describing the dynamic and collaborative nature of a whole-class discussion in the lesson. The term "Neriage" in Japanese refers to "kneading up" or "polishing up". In the context of teaching, the term works as a metaphor for the process of "polishing" students' ideas and getting an integrated mathematical idea through a whole-class discussion. Japanese teachers regard "Neriage" as critical for the success or failure of the entire lessons.

Based on his or her observations during "Kikan-shido", the teacher carefully calls on students, asking them to present their methods of solving the problem on the chalkboard, selecting the students in a particular order. The order is quite important to the teacher both for encouraging those students who found naive methods and for showing students' ideas in relation to the mathematical connections that will be discussed later. In some cases, even an incorrect method or error may be presented, if the

teacher thinks it would be beneficial for the class. Students' ideas are presented on the chalkboard, to be compared with each other with oral explanations. The teacher's role is not to point out the best solution but to guide the discussion by the students towards an integrated idea.

"Matome" as summing up

"Matome" in Japanese means "summing up". Japanese teachers think that this stage is indispensable to any successful lesson. It is identified as a critical difference between the U.S. and Japanese classroom activities (Fujii, et al., 1998). According to the U.S.-Japan comparative analysis, at the Matome stage Japanese teachers tend to make a final and careful comment on students' work in terms of mathematical sophistication.

Generally speaking, in the Matome stage what students have discussed in the whole-class discussion is reviewed briefly and what they have learned through the lesson is summarized by the teacher.

Some practical ideas shared by Japanese teachers.

Ensuring the Student's "Ownership"

During the discussion, each solution method is labeled with the name of student who originally presented it. That is, the name of student who presented the solution will be written, or a small magnet card with his/her name will be put on the chalkboard. Thus, each solution method is referred using the name of student in the discussion. This practical technique may seem to be trivial but it is very important to ensure each student's "ownership" of presented methods.

"Bansho": Effective Use of Chalkboard

Another important technique used the teacher relates to the use of chalkboard, which is referred as "Bansho" by Japanese teachers. Teachers usually try to keep all that is written during the lesson on the chalkboard without erasing if possible. From the learner's perspective, it is easier to compare multiple solution methods if

they appear on the chalkboard simultaneously. Also, the chalkboard can be a written record of the entire lesson, which gives both the students and teacher a birds-eye view of what has happened in the class at the end of each lesson.

Teaching and Evaluation as Two Faces of the Same Coin

Teachers are conducting formative evaluations during their lessons to obtain instantaneous feedback on their instructional techniques. Such evaluations are embedded in each teacher's role described above.

When the teacher moves about the classroom for "Kikan-shido" during problem solving by students, he or she is monitoring students' activities silently to assess their status or making suggestions to individual students who need help or guidance. Thus, it is important to see integrating teaching and its evaluation as two faces of the same coin.

The Importance of Evaluation as Incorporated with Teaching

- Teaching and evaluation activities are done to ensure that the teaching goals established based on the curriculum and teaching plans are being achieved by the students with whom the teacher is currently working.
- Teacher evaluations are intended to enhance teaching practices. For example, they can allow teachers to ascertain the effectiveness of their teaching practices and help them improve their teaching plans by incorporating the results into their teaching.
- For students, evaluations are an important tool for making them aware of how well they are learning, giving them an opportunity to adjust their behaviors, and enabling them to set their own learning goals.
- Incorporating teaching and evaluation in the teaching process makes it possible to plan comprehensive evaluations that look at both the teaching process and its results.

Section 6.2: How do Japanese Teachers Evaluate Their Students in Their Lessons?

Hiroyuki Ninomiya

Teachers are conducting formative evaluations during their lessons to obtain instantaneous feedback on their instruction techniques. These evaluations may be completed by students individually or in groups. Let us remind at first general image of the Japanese classroom. To get the general idea, we gather together information already explained in the text, add some comments and give actual examples.

Image of the Japanese Classroom (*The Teaching Gap*, Stigler & Hiebert, 1999)

1. Teacher reviews previous lesson and assigns a problem that was not finished.
2. Students present solution methods they have found, and teacher summarizes.
3. Teacher presents task for the day and asks students *to work on it independently* (task is to invent problem for classmates to solve).
4. Teacher instructs students to work in small groups. Leaders of groups share problems with teacher, who writes them on board. Students copy problems and begin working on them.
5. Teacher highlights a good method for solving these problems.

The Objectives of Desk Instruction

“Desk Instruction” occurs when a teacher walks around between student desks while students are working individually to examine

students' learning and to provide help or guidance as needed. Desk instruction has two objectives. The first is to ascertain how well individual students are learning (their level of recognition) so as to create more active argumentative communication and deepen group thinking during the presentation period after the individual study period, and to formulate discussion ideas so that a diversity of opinions can be expressed during the group learning period. The second objective is to help eliminate individual errors and to improve students' academic abilities. Therefore, following questions should be considered to make clear the objectives of desk instruction (Table 1).

Table 1: Are the objectives of desk instructions clear to you?

- | |
|---|
| <ul style="list-style-type: none">• To know the level of students' understanding?• To ascertain types of reactions (as preparation to choose students who will present information)?• To support students who have trouble learning?• To support students' group activities? |
|---|

Integration of Teaching and Evaluation

Teaching and evaluation activities are done to ensure that the teaching goals established based on the curriculum and teaching plans are being achieved by the students with whom the teacher is currently working.

For teacher, evaluations are intended to enhance teaching practices. For example, they can allow teachers to ascertain the effectiveness of their teaching practices and help them improve their teaching plans by incorporating the results into their teaching.

For students, evaluations are an important tool for making them aware of how well they are learning, giving them an opportunity to

adjust their behaviors, and enabling them to set their own learning goals.

Incorporating teaching and evaluation in the teaching process makes it possible to plan comprehensive evaluations that look at both the teaching process and its results. Hence, two senses of the integration of teaching and evaluation are used as in what follows.

Two Senses of the Integration of Teaching and Evaluation

1. Integration in the sense of using the evaluation results in future teaching development efforts and teaching plans: Evaluations should not be performed at the end of a teaching activity, but during the activity. This way the teacher can use the results to examine and make adjustments to the teaching practices they have used thus far, and can either adopt new or supplementary teaching practices. There should be an emphasis on formative evaluations.
2. Integration in the sense of using the evaluation process itself as a teaching tool: Evaluations serve as a means of teaching students. For example, an evaluation that tells the student they “worked really hard” simultaneously helps stimulate the student’s desire to learn. Below are some examples of a range of rich evaluative comments that Japanese teachers use to encourage students, to provide constructive feedback to assist learning, and to foster more effective participation in mathematics classrooms (Table 2).

Table 2: Some examples of evaluative feedback between teachers and students

<p>Verbal Assessments which value students' endeavor</p>	<p>You have improved a lot, as you always try to think deeply. You have concentrated a lot. Wonderful! Great! You could solve such many problems beautifully.</p>
<p>Verbal Assessments which make students try harder</p>	<p>It's a pity, but it is OK. You can do it. Try better next time. You can do it well if you apply yesterday's outcomes. You had better try another angle. Be confident!</p>
<p>Verbal Assessments which stimulate students' interest and motivation</p>	<p>You have been working with a lot of confidence. Now it seems you like to learn about fractions. Yours is such a good question. It interests everyone.</p>
<p>Verbal Assessments which value students' ability</p>	<p>Since you have understood his idea so well, please explain it to everyone. Wow, you are the champion of multiplication! Your explanation is very clear and really helpful to understand.</p>
<p>Verbal Assessments which gives energy and hope for learning</p>	<p>You seemed to think you could not understand today's problem. Ok, I will work with you tomorrow until you can be satisfied with your understanding. You had made a lot of careless mistakes because you were in a rush, but now you have very few. You are thinking deeply and more carefully.</p>
<p>Verbal Assessments which value students' contributions</p>	<p>Because of your question, we got some good hints for solving this problem. Because you carefully explained your idea, many people could understand quite a lot.</p>